

**Pre-Calculus 12 Session 9  
Tuesday, February 8, 2022**

1. Last Day's Homework:
  - Practice: Section 3.2: pages 124-125, Practise 1, 2, 3a, 4c, 5b, 6a, c, e, 7a, c, 8a,c, 10, 12.
  - Readings: Nothing new.
  - Hand-in Assignments and other things: The Chapter 3 Hand-in Assignment will be due on THURSDAY, FEBRUARY 10. The Chapter 3 Test will be on Tuesday, February 15.
2. A Little more about Section 3.4: Solving Equations of, and Sketching Graphs of, Polynomial Functions, Including Sketching Graphs of Polynomials using Transformations
3. Section 4.1: Angles and Angle Measures
4. Section 4.2: The Unit Circle
5. Section 4.3: Trigonometric Ratios

**Homework:** This depends on how far we get today.

**Readings:** Nothing new.

**Practice from Textbook to try:**

- Section 3.4: pages 147-150, Practise 1, 2a, c, e, 4a, c, 5, 7, 8a, c, 9a, c, e, 10a, c, 11, 14, 16.
- Section 4.1: pages 175-176, Practise 1, 2a), c), e), 4a), c), e), 6a), c), e), 7a), c), e), 8a), c), 9a), b), 12a), c), 13.
- Section 4.2: pages 186-188, Practise 2a), c), e), 2a), c), e), 3a), c), 4a), c), e), h), i), 5a), c), e), 6, 7, 9, 13, 18.
- Section 4.3: pages 201-203, 1a), c), e), g), i), k), 2a), c), e), g), i), k), 3a), c), e), 6a), c), e), 9a), c), e), 10 (all parts), 11 (all parts), 12a), c).

**Hand-in Assignments:** Continue working on the Chapter 3 Hand-in Assignment. NOTE: It will be due in next class. Additionally, you should begin working on the Chapter 4 Hand-in Assignment. That assignment will likely be due in on Thursday, February 17.

**The Chapter 3 Test will be on Tuesday, February 15.**

To be properly prepared for the Chapter 1 Retest, the Chapter 3 Test, and all future tests, you need to be attempting all the suggested textbook Practise. Additionally, you should attempt the Chapter Practice Tests from the textbook (e.g. the Chapter 1 Practice Test on pages 58 and 59 of the textbook).

Ch 3 Practice test pp 151-152  
Review pp 153-154.

Section 3.3: Factoring

When all other factoring methods fail, you need to use the integral zero theorem to determine

The possible integral zeros of the corresponding function of the polynomial.

eg  $x^3 + 2x^2 - 13x + 10$

possible integral zeros of the corresponding function are the factors of 10

i.e.  $\pm 1, \pm 2, \pm 5, \pm 10$

- Rather than check the 8 possibilities, use your calculator to find one of the zeros. Then divide to get the depressed polynomial and factor that using appropriate methods.

So, input  $Y_1 = x^3 + 2x^2 - 13x + 10$  into a graphing calculator. Then go to the TABLE Graph

X	Y1
-5	-56
-4	0
-3	40
-2	24
-1	10

X = -5

Since -1 is a zero of  $P(x) = x^3 + 2x^2 - 13x + 10$ ,  $(x + 5)$  is a factor of

$$\begin{array}{r}
 x^2 - 3x + 2 \\
 \hline
 x + 5 \ ) \ x^3 + 2x^2 - 13x + 10 \\
 \underline{-(x^3 + 5x^2)} \phantom{+ 10} \\
 -3x^2 - 13x \phantom{+ 10} \\
 \underline{-(-3x^2 - 15x)} \phantom{+ 10} \\
 2x + 10 \\
 \underline{-(2x + 10)} \\
 0
 \end{array}$$

Factor me!!

$(x - 1)(x - 2)$

1 and 2 are also zeros of  $P(x) = x^3 + 2x^2 - 13x + 10$

$\therefore x^3 + 2x^2 - 13x + 10 = (x + 5)(x - 1)(x - 2)$

It's important to be able to factor high degree

polynomials so that we can

1) sketch graphs of polynomial functions  
without a graphing calculator.

→ important features are:

- end behaviour
- zeros (x-axis intercepts)
- y-axis intercept (constant term)

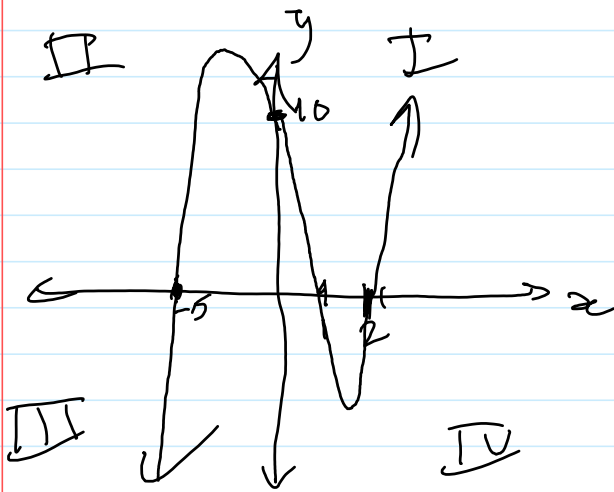
$$P(x) = x^3 + 2x^2 - 13x + 10 = (x+5)(x-1)(x-2)$$

Zeros are at  $x = -5, 1, 2$

y-axis intercept =  $y = 10$   $(0, 10)$

End Behaviour: Start down in  $Q_{III}$

ends up in  $Q_I$



## Multiplicity of zeros

If the multiplicity of a zero is 1, the graph-line passes through that zero (x-intercept).

If the multiplicity is 2, 4, 6, ... or any other even number, the graph-line has a point of tangency at that point.

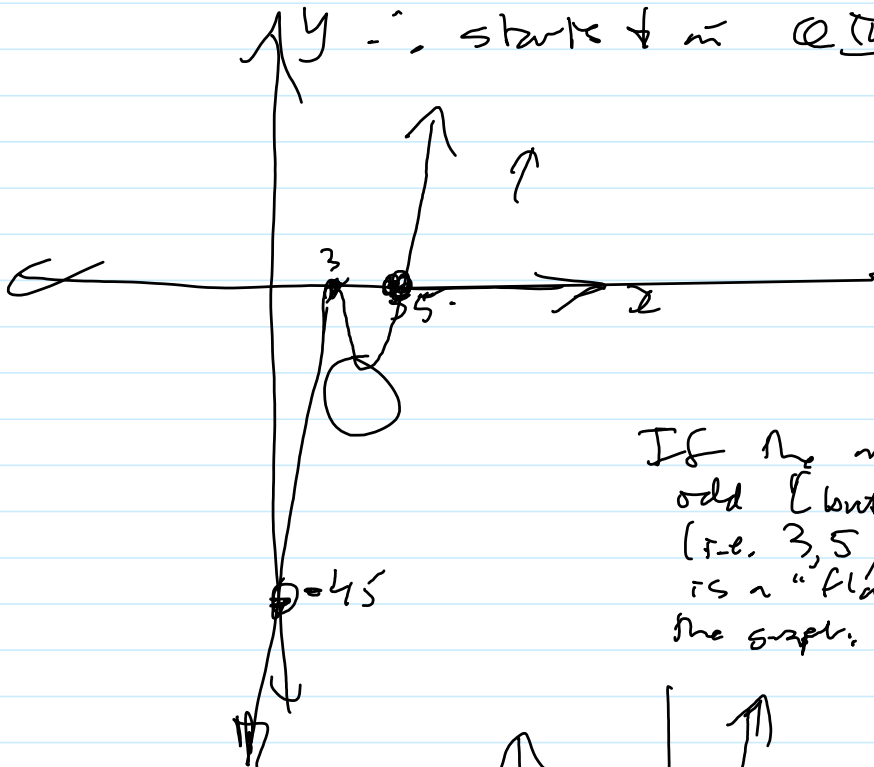
$$P(x) = (x-3)^2(x-5) = (x-3)(x-3)(x-5)$$

zeros:  $x = 3$  (multiplicity of 2)  $-3 \times -3 \times -5$   
 $x = 5$  (multiplicity of 1)  $-45$

There's a point of tangency at  $x = 3$

Constant term =  $(-3 \times -3 \times -5) = -45 = y$ -axis

- it's a cubic with a positive leading coefficient.  
 intercept.  
 $y$  - starts  $\downarrow$  in  $Q(IV)$ , ends  $\uparrow$  in  $Q I$



If the multiplicity is odd (but not 1), (i.e. 3, 5, 7, 9...), there is a "flat spot" in the graph.



The other reason why we want to factor polynomials is so that we can solve higher degree polynomial equations.

$$P(x) = x^3 + 2x^2 - 13x + 10 = (x-1)(x+5)(x-2)$$

- This function has zeros at  $x=1$ ,  $-5$  and  $2$

The corresponding equation is

$$x^3 + 2x^2 - 13x + 10 = 0$$

The roots of this equation can be easily determined if the equation is written in factored form.

$$(x-1)(x+5)(x-2) = 0$$

If any factor equals zero, the product of the factors is zero.

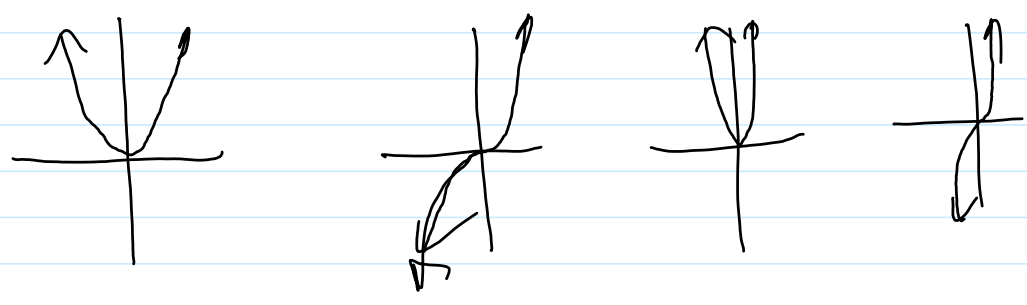
$x(x-1)(x+5)(x-2) = 0$  ... any factor equals zero, the product of the factors equals zero.

$x-1=0 \Rightarrow x=1$       $x+5=0 \Rightarrow x=-5$       $x-2=0 \Rightarrow x=2$

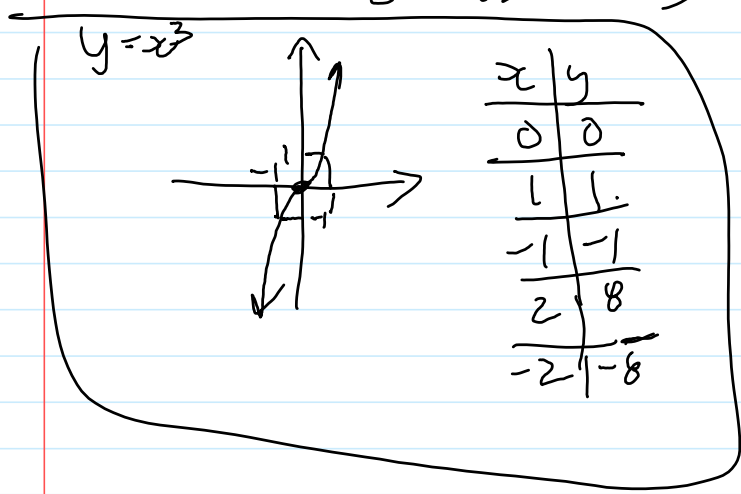
The roots of an equation are equal to the zeros of the corresponding function.

We can graph polynomial functions using transformations of the "base" polynomials

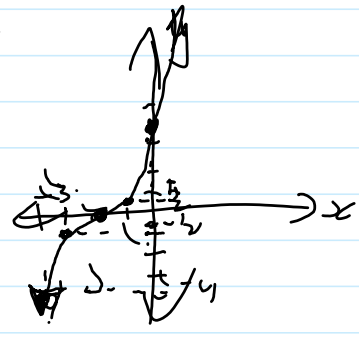
$y = x^2$       $y = x^3$       $y = x^4$       $y = x^5$



Sketch  $y = \frac{1}{2}(x+2)^3$   $\Leftarrow$  UC by  $\frac{1}{2}$ , 2 units left.



x	y
-2	0
-1	1/2
-3	-3/2
0	4
-4	-4



Chapters 4, 5 and 6: Trigonometry  $\geq \frac{1}{3}$   
of course!

Chapter 4: Trigonometry and the Unit Circle

3  
of course!

A Bit of Review.

You should have studied right triangle  
trigonometry