PC 12 Session 8

February 2, 2022 4:31 PM

Pre-Calculus 12 Session 8
Thursday, February 3, 2022

1. Last Day's Homework:
   - Practice: Section 3.2: pages 124-125, Practise 1, 2, 3a, 4c, 5b, 6a, c, e, 7a, c, 8a,c, 10, 12.
   - Readings: Nothing new.
   - Hand-in Assignments and other things: The Chapter 3 Hand-in Assignment will be due on ~~TUESDAY, FEBRUARY 8~~ (depending whether or not we finish Chapter 3 today). The Chapter 3 Test will likely be on Thursday, February 10 (same condition as above).

   Thursday, Feb 10, test Tuesday, Feb 15..
2. More about Section 3.3: The Factor Theorem and Fun with Factoring!
3. Section 3.4: Solving Equations of, and Sketching Graphs of, Polynomial Functions, Including Sketching Graphs of Polynomials using Transformations
4. Section 4.1: Angles and Angle Measures Trigonometry!!
5. Section 4.2: The Unit Circle

Homework: This depends on how far we get today.

Readings: Nothing new.

Practice from Textbook to try:

Section 3.3: pages 133-135, Practise 1, 2a, c, e, 3a, c, e, 4a, c, e, 5a, c, e, 6a, e, 7b, d, 9, 11, 14.
Section 3.4: pages 147-150, Practise 1, 2a, c, e, 4a, c, 5, 7, 8a, c, 9a, c, e, 10a, c, 11, 14, 16.
Section 4.1: pages 175-176, Practise 1, 2a), c), e), 4a), c), e), 6a), c), e), 7a), c), e), 8a), c), 9a), b), 12a), c), 13.
Section 4.2: pages 186-188, Practise 2a), c), e), 2a), c), e), 3a), c), 4a), c), e), h), i), 5a), c), e), 6, 7, 9, 13, 18.

Hand-in Assignments: Continue working on the Chapter 3 Hand-in Assignment. NOTE: It will be due in on Tuesday, February 8 (same condition mentioned above).

The Chapter 3 Test will likely be on Thursday, February 10. This will depend on how far we get today and next day.

To be properly prepared for the Chapter 1 Retest, the Chapter 3 Test, and all future tests, you need to be attempting all the suggested textbook Practise. Additionally, you should attempt the Chapter Practice Tests from the textbook (e.g. the Chapter 1 Practice Test on pages 58 and 59 of the textbook).

Last day: The Remainder Theorem

· if a polynomial $P(x)$ is divided by a binomial

$x+b \to x-(-b)$

$$P(x) = x^3 + 2x^2 - 5x - 7$$
$$\div (x+3)$$
$$x - -3$$
$$= P(-3) = (-3)^3 + 2(-3)^2 - 5(-3) - 7$$
$$= -27 + 18 + 15 - 7 = -1$$

Factoring Methods

1) Common factoring.

$2x+4 = 2(x+4)$

$x^3 + 2x = x(x^2+2)$ ← can this be factored? NO

2) Difference of perfect squares

$x^2 - 4 = (x+2)(x-2) = x^2 - \cancel{2x} + \cancel{2x} - 4$
$(x)^2 - (2)^2$

$= x^2 - 4$

$9x^2 - 16$
$(3x)^2 - (4)^2 = (3x+4)(3x-4)$

$16x^4 - 81$
$(4x^2) - (9)^2 = (4x^2+9)(4x^2-9)$ this can be factored
$(2x)^2 - (3)^2$

Sum of squares does not factor.

$= (4x^2+9)(2x+3)(2x-3)$

Simple
Quadratic Trinomials

$x^2 - 3x + 2 = (x-1)(x-2)$

$\boxed{-1} \times \boxed{-2} = {}^{+}2$

$\boxed{-1} + \boxed{-2} = -3$

$x^2 - 5x + 6 = (x-2)(x-3)$

$\boxed{-2} \times \boxed{-3} = 6$ $\quad$ $+6 = -1 \times -6$
$\boxed{} + \boxed{} = -5$ $\quad$ $+1 \times +6$
-2×-3 ✓
$+2 \times +3$

$1x^2 + 4x + 4 = (x+2)(x+2)$
a, c

$= (x+2)^2$

More complicated trinomials $(a \neq 1)$

If $a \neq 1$, factoring gets a bit more difficult.

- I use the method of decomposition

$ax^2 + bx + c$ multiply $a \times c$
look for 2 factors of that product
which add to b.

Sums and differences of cubes

$$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

$$x^3 - 27 = (x-3)(x^2 + 3x + 9)$$
$$x^3 - (3)^3$$

$$8x^3 + 125 = (2x+5)(4x^2 - 10x + 25)$$
$$(2x)^3 + (5)^3$$
$$(2x)^2 = 4x^2$$

Factoring by grouping

If you have 4 terms, you could try "factoring by grouping".

Q: What if you have a higher degree polynomial that can't be group factored!

eg. $P(x) = x^3 + 6x^2 - 7x - 60$

We need to find a binomial factor of form $(x-a)$ first. Once we find one, we can divide it into $P(x)$ to get the quotient. Then we factor the quotient.

The integral zeros theorem tells us that if $x-a$ is a factor of $P(x)$, then "a" must be a factor of the constant term of $P(x)$!

In the above, -60 is the constant term.

Factors of -60 are:

$-1, +1, -2, +2, -3, +3, -4, +4, -5, +5, -6, +6, -10, +10,$

$-12, +12, -15, +15, -20, +20, -30, +30, -60, +60.$

possible factors of $P(x)$ are:

$(x+1), (x-1), (x+2), (x-2), (x+3), (x-3), \ldots$

$(x+60), (x-60)$

$P(x) = x^3 + 6x^2 - 7x - 60$

$P(1) = 1 + 6 - 7 - 60 =$ $\qquad$ $P(2) =$

$P(-1) = -1 + 6 + 7 - 60$ $\qquad$ $P(-2) =$

$P(3) = 27 + 54 - 21 - 60$
$= 81 - 81 = 0$ ✓✓

$\therefore x - 3$ is a factor for $P(x)$

<u>Next</u> use long $\div$ or synthetic $\div$ to find the quotient

of $P(x) \div (x-3)$

$$\begin{array}{r|rrr|r} -3 & 1 & 6 & -7 & -60 \\ - & \downarrow & -3 & -27 & -60 \\ \hline x & 1 & 9 & 20 & 0 \end{array}$$

$1x^2 + 9x + 20 \leftarrow$ factor this?

$(x+4)(x+5)$

$\therefore P(x) = x^3 + 6x^2 - 7x - 60 = (x-3)(x+4)(x+5)$
$\qquad x = 3 \quad x = -4 \quad x = -5$

$\therefore$ the zeros of $P(x)$ are $x = 3,\ x = -4,\ x = -5$

<u>Multiplicity of factors</u>

Some polynomials can have two or more identical factors.

$f(x) = x^4 + x^3 - 10x^2 - 4x + 24$

$= (x+3)(x+2)(x-2)^2$

We see the factor $(x-2)$ has multiplicity of 2.

We say the factor $(x-2)$ has multiplicity of 2.

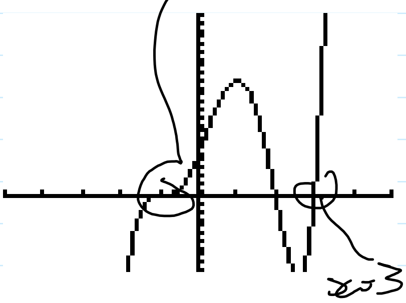
Also, we say that the zero of $x=2$ has multiplicity of 2

$P(x) = (x-2)(x+1)^3(x-3)$

$(x+1)$ has multiplicity of 3.

$y=(x-1)^5$

Zero at $x=-1$

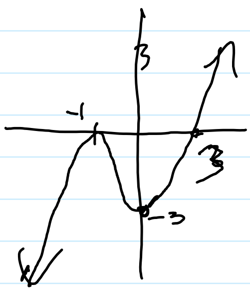


There's a "flat spot" at $x=-1$.

This is called an inflection point.

If the multiplicity of a zero is 1, the graph crosses through the x-axis at that zero.

If the multiplicity of a zero is an even number (2, 4, 6 ...), there's a "point of tangency".



$y=(x+1)^2(x-3) = (x+1)(x+1)(x-3)$

Constant term $= 1 \times 1 \times -3 = -3$