PC 12 Session 7

February 1, 2022 9:24 AM

Pre-Calculus 12 Session 7
Tuesday, February 1, 2022

1. Last Day's Homework:
   - Practice: Section 3.1: page 114, Practise 1, 2, 6, 7, 9.
   - Readings: Section 4.2 (pages 180 to 186), Section 4.3 (pages 191 to 201), Section 4.4 (pages 206 to 211).
   - Hand-in Assignments and other things: The Chapter 3 Hand-in Assignment will be due on TUESDAY, FEBRUARY 8. The Chapter 3 Test will likely be on Thursday, February 10. (maybe Tuesday, Feb 15).
2. Return of, and Many Comments on, the Chapter 1 Test
3. Section 3.2: Division of Polynomials by Binomials, Synthetic Division and the Remainder Theorem
4. Section 3.3: The Factor Theorem and Fun with Factoring!
5. Section 3.4: Solving Equations of, and Sketching Graphs of, Polynomial Functions

Homework: This depends on how far we get today.

Readings: Nothing new.

Practice from Textbook to try:

Section 3.2: pages 124-125, Practise 1, 2, 3a, 4c, 5b, 6a, c, e, 7a, c, 8a,c, 10, 12.
Section 3.3: pages 133-135, Practise 1, 2a, c, e, 3a, c, e, 4a, c, e, 5a, c, e, 6a, e, 7b, d, 9, 11, 14.
Section 3.4: pages 147-150, Practise 1, 2a, c, e, 4a, c, 5, 7, 8a, c, 9a, c, e, 10a, c, 11, 14, 16.

Hand-in Assignments: Continue working on the Chapter 3 Hand-in Assignment. NOTE: It will be due in on Tuesday, February 8.

The Chapter 3 Test will likely be on Thursday, February 10. This will depend on how far we get today and next day.

To be properly prepared for the Chapter 1 Retest, the Chapter 3 Test, and all future tests, you need to be attempting all the suggested textbook Practise. Additionally, you should attempt the Chapter Practice Tests from the textbook (e.g. the Chapter 1 Practice Test on pages 58 and 59 of the textbook).

About Ch. 1 Retest

→ must be written outside of class-time

→ best times for me: M/W 2:45–5:45 (here).

- other times are available. e.g. Tues 7:30 – 9:30 (with Neha)
(advanced planning required).

- I will take the higher of the 2 test scores, unless you had 90% or more on the first test. In such cases, I will average your 2 test marks

---

The returned tests are yours to keep.

The returned tests are yours to keep.

I would like all retests completed by class time on Thursday, Feb 10.

If you are planning to do the retest, you still need to keep up with the new course material!!

Last day

Long Division of a polynomial by a binomial.

More examples (see Polynomial Note package)

Synthetic Division

Recall:

Previously, we did this using long division.

$$(x^3 - 12x^2 - 42) \div (x - 3) = (1x^2 - 9x - 27) + \frac{-123}{x-3}$$

R

$$1x^3 - 12x^2 + 0x - 42$$

Here's another way to do it.

−3	1	−12	0	−42	
−		−3	+27	+81	
X	1	−9	−27		−123

- Bring down the 1st coefficient
- multiply and then subtract.
- repeat ...

$Q(x) = 1x^2 - 9x - 27$ remainder $= -123$

The Remainder Theorem

When dividing a polynomial $P(x)$ by a binomial having the form $(x - a)$, the value of the remainder can tell is something very significant.

Thus far, we've seen two ways of determining the remainder when a polynomial is divided by a binomial.

Long Division — Synthetic Division.

$$x-(-3)\overline{\big)\,4x^4+3x^3-7x^2-2x+5}$$

$\vdots$

$$\begin{array}{r|rrrrr} -3 & 4 & 3 & -7 & -2 & 5 \\ & & & & & \\ \hline & & & & & \boxed{} \end{array}$$

There's another way to determine the remainder of these divisions!

If a polynomial, $P(x)$, is divided by a binomial of form $(x-a)$, the remainder is equal to $P(a)$.

5a) $\underbrace{(4w^4+3w^3-7w^2+2w-1)}_{P(w)} \div (w+2)$

$(w+2) = (w-(-2))$

$$P(-2) = 4(-2)^4 + 3(-2)^3 - 7(-2)^2 + 2(-2) - 1$$
$$= 4(16) + 3(-8) - 7(4) + 2(-2) - 1$$
$$= 64 - 24 - 28 - 4 - 1 = 7$$

Important: if the divisor has the form $(x-a)$ then $P(a) =$ remainder.

If there's a $+$ sign in the binomial, you need to change it to a $-$ which means the sign of the constant term must change.

eg. $(x+6) \longrightarrow (x-\underbrace{(-6)}_{a})$

<u>The Factor Theorem</u>

The Factor Theorem

If a polynomial $P(x)$ is divided by a binomial of form $(x-a)$ and the remainder $= 0$, then $(x-a)$ is a factor of $P(x)$.

$$P(x) = Q(x) \times (x-a) + \cancel{R}^{\,0}$$
$$= (x-a)(Q(x)).$$

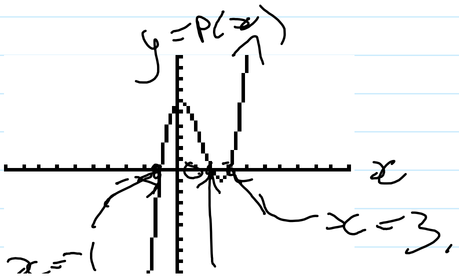
When $P(x) = x^3 - 4x^2 + x + 6$ is divided by $(x+1)$ or $(x-2)$ or $(x-3)$ the remainder $= 0$.

$\therefore$ $(x+1)$, $(x-2)$ and $(x-3)$ must ALL be factors of $P(x)$.

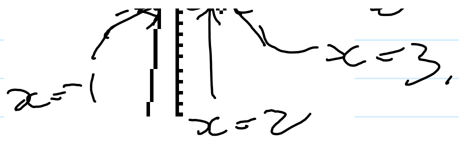
i.e. $$x^3 - 4x^2 + x + 6 = \boxed{(x+1)(x-2)(x-3)}.$$
$$= (x^2 - x - 2)(x-3)$$
$$= x^3 - 3x^2 - x^2 + 3x - 2x + 6$$
$$= x^3 - 4x^2 + x + 6 \quad ✓✓.$$

The zeros of a polynomial, $P(x)$, are the values of x that make $P(x) = 0$

Here, we saw that $P(-1)$, $P(2)$ and $P(3)$ are all equal to zero, $\therefore$ $x=-1$, $x=2$ and $x=3$ are the zeros of $P(x) = x^3 - 4x^2 + x + 6$.



The zeros of a polynomial are the same as the x-intercepts of the graph of that polynomial



x-intercepts of the graph of that polynomial.