PC 12 Session 5

January 23, 2022 11:07 AM

"Function or not a Function" → error!!

Pre-Calculus 12 Session 5
Tuesday, January 25, 2022

1. Last Day's Homework: Section 1.3: pages 38 to 40, Practise 4, 5a, 6, 7a, b, c, d, 8, 9c, e, 10a, b, Section 1.4: pages 51 to 54, Practice 1b, 2a, 3a,c, 5a, e, 8b, 12a.

The Unit 1 Hand-in Assignment is due in today.

2a) A Little Bit More on Section 1.3: Combining Transformations and Section 1.4: Inverses of Functions and Relations

2b) Q&A about

3. Section 3.1: Characteristics of Polynomial Functions and their Graphs

4. Section 3.2: The Remainder Theorem

5. Section 3.3: The Factor Theorem and Fun with Factoring!

6. Section 3.4: Equations and Graphs of Polynomial Functions (should we get this far)

Homework: This depends on how far we get today.

Readings: Section 3.4 (pages 136 to 147), Section 4.1 (pages 166 to 175).

Practice from Textbook to try:

Section 3.1: page 114, Practise 1, 2, 6, 7, 9.
Section 3.2: pages 124-125, Practise 1, 2, 3a, 4c, 5b, 6a, c, e, 7a, c, 8a,c, 10, 12.
Section 3.3: pages 133-135, Practise 1, 2a, c, e, 3a, c, e, 4a, c, e, 5a, c, e, 6a, e, 7b, d, 9, 11, 14.
Section 3.4: pages 147-150, 1, 2a, c, e, 4a, c, 5, 7, 8a, c, 9a, c, e, 10a, c, 11, 14, 16.

Hand-in Assignments: Continue working on the Chapter 3 Hand-in Assignment. It will likely be due in on Tuesday, February 1.

The Chapter 1 Test is next day. To help prepare for this, you may want to work through the Chapter 1 Review and the Chapter 1 Self-Test on pages 56 to 59 of the text.

The Chapter 3 Test will likely be on either Thursday, February 3, or Tuesday, February 8. It will depend on how far we get today and next day.

A Bit More About Inverses of Relations

- To get the inverse of a relation, we reflect the relation over the line $y = x$.
  - this simply switches the x & y values around.
- Writing the equation of the inverse relation requires

that you:

1) write the equation of the relation in the form $y = f(x)$

2) Switch x and y around
3) Solve algebraically for y.

$f(x) = 3x - 5$

$y = 3x - 5$

$x = 3y - 5$
$+5 \qquad +5$

$\frac{x+5}{3} = \frac{\cancel{3}y}{\cancel{3}}$

$y = \frac{x+5}{3} \quad \therefore \quad f^{-1}(x) = \frac{x+5}{3}$

$f(x) = 2x^2 - 3$

$y = 2x^2 - 3$

$x = 2y^2 - 3$
$+3 \qquad +3$

$\frac{x+3}{2} = \frac{\cancel{2}y^2}{\cancel{2}}$

$\pm\sqrt{\frac{x+3}{2}} = \sqrt{y^2}$

$y = \pm\sqrt{\frac{x+3}{2}} \qquad f^{-1}(x) = \pm\sqrt{\frac{x+3}{2}}$

Inverses of Rational functions
have x terms in the denominator.

$y = \frac{1}{x} \qquad y = \frac{2x+1}{4x-7}$

- if there's x-terms in the numerator AND the denominator, things get tough.

$y = \frac{2x+1}{4x-7}$ Switch x and y
this $\neq 0 \quad \therefore \ 4x-7 \neq 0 \quad 4x \neq 7$ $\quad x \neq \frac{7}{4}$ restriction on x.

$x = \frac{2y+1}{4y-7}$ mult. both sides by the denominator

$x(4y-7) = \frac{2y+1}{\cancel{4y-7}} \times \cancel{4y-7}$

$x(4y-7) = 2y+1$ expand the LHS

$4xy - 7x = 2y + 1$ gather y-terms to the same side
$-2y \qquad -2y$

$$4xy - 7x = 2y + 1$$
$$\quad -2y \qquad -2y$$

gather y-terms to the same side

$$4xy - 2y - 7x = 1$$
$$\quad +7x \qquad +7x$$

- any "x only" terms go the other side.

$$4\underline{xy} - 2\underline{y} = 1 + 7x$$

- factor out the y on the LHS

$$\frac{y(\cancel{4x-2})}{\cancel{4x-2}} = \frac{1+7x}{4x-2}$$

· ÷ both sides to isolate the y.

$$y = \frac{1+7x}{4x-2} \qquad f^{-1}(x) = \frac{1+7x}{4x-2}$$

$$4x - 2 \neq 0$$
$$4x \neq 2$$
$$x \neq 2/4 = \tfrac{1}{2}$$
$$x \neq \tfrac{1}{2}.$$

Chapter 3: Polynomials

One of the things we will be doing a lot of is graphing polynomial functions.

- The graphs of polynomial function can have all sorts of appearances.

→ we will look at some key features of the graphs of polynomial functions of various degrees.

End behavior = describes where the graphs start and end, and the direction, as we go from left to right.

Set A $\quad y = x^1, y = x^2, y = x^3, y = x^4, y = x^5$

Either the graph starts down in QIII and ends up in QI OR it starts up in QII and ends up in QI..

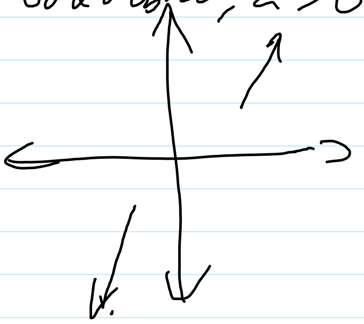
If the leading coefficient is + and the degree is ODD, the end behaviour (EB) is: starts ↓ in QIII, ends up in QI.

If the leading coefficient is negative − and the degree is ODD, the end behaviours is: starts ↑ in QII, ends ↓ in QIV

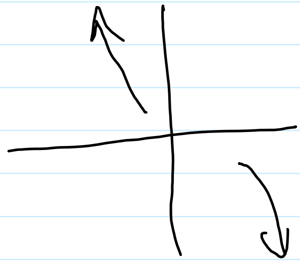
If the leading "a" coefficient is + and the degree is EVEN, the EB is: starts up in QII, ends up in QI.

If the leading coefficient is − and the degree is EVEN, the EB is: starts ↓ in QIII, end ↓ in QIV

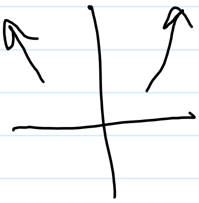
odd degree, $a > 0$



odd degree, $a < 0$



even degree, $a > 0$

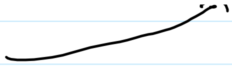


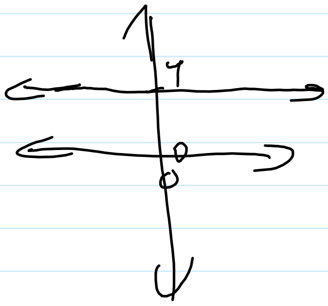
even degree, $a < 0$

What about zero degree polynomials?

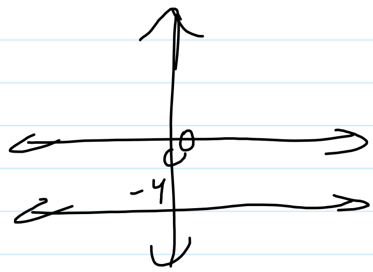
$y = \text{constant}$ eg $y = 4x^0$ ← leading coefficient

$\underline{y = 4}$

$y = 4$  - leading coefficient



if $y = -4$



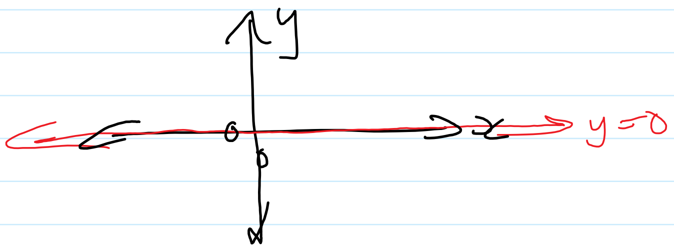
I II III

IV V
5
VI
6
VII
VIII
IX 9
X = 10

+ constant
- extends horizontally from Q II to Q I

− constant
- extends horizontally from Q III to Q IV

What about $y = 0$?



· Extends horizontally along the x-axis.

All the function in Set A have 1 x-intercept (at $x=0$) and 1 y-intercept (at $y=0$).

- They all have a leading coefficient of (+)1
- They all have a constant term equal to zero.
($y = x + 0$, $y = x^2 + 0$ etc.).

- Number of turning point (TP's)

 A TP is a change in direction as we go left to right.

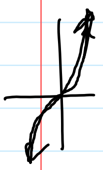
$y = x^1$ # of TP's = 0
D: $\{x \mid x \in \mathbb{R}\}$
 R: $\{y \mid y \in \mathbb{R}\}$
$y = x^2$ # of TP's = 1
D: $\{x \mid x \in \mathbb{R}\}$

$y = x^4$ # of TP's = 1
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$
$y = x^5$ # of TP's = 0 

$y = x^2$ # of TP's = 1

$D: \{x \mid x \in \mathbb{R}\}$

$R: \{y \mid y \geq 0, y \in \mathbb{R}\}$



$y = x^3$: # of TP's = 0

$D: \{x \mid \in \mathbb{R}\}$

$R: \{y \mid y \in \mathbb{R}\}$

$y = x^0$ # of TP's = 0

$D: \{x \mid x \in \mathbb{R}\}$

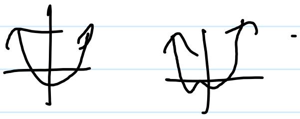
$R: \{y \mid y \in \mathbb{R}\}$

The domain of <u>ALL</u> polynomial functions is $\{x \mid x \in \mathbb{R}\}$ (all real numbers).

If the degree is <u>ODD</u>, the range is:

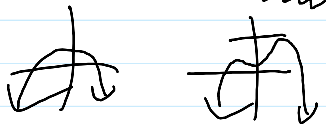
$\{y \mid y \in \mathbb{R}\}$

However, the range of the EVEN degree functions is <u>limited</u>.



Leading coeff is +: $R: \{y \mid y \geq \text{minimum}, y \in \mathbb{R}\}$ opens up

" " is −: $R: \{y \mid y \leq \text{maximum}, y \in \mathbb{R}\}$ opens down.



FOR <u>ALL</u> polynomials, the y-intercept is equal to the constant term. (There's only 1 y-int)

Next day: more about # of <u>x-intercepts, & # of TP's</u>.

Test.