PC 12 Session 3

January 18, 2022 9:13 AM

Pre-Calculus 12 Session 3
Tuesday, January 18, 2022

1. Have you submitted your Student Information Sheet? Your completed Daily Health Check Form? Anybody new?

2. Last Day's Homework: Section 1.1: pages 12 to 14, Practise 2, 3c,d, 4a,c, 5, 8, 11 (although we did not finish Section 1.1)

3. More about Section 1.1: Horizontal and Vertical Translations of Functions
   - More about Horizontal and Vertical Translations of Graphs of Functions, Mapping Notation, and the Equations of Horizontally Translated Functions
   - Combined Translations and Translations of already Translated Functions
   - The Coordinates of Translated Points
   - Translations Review

4. Section 1.2: Reflections, Expansions (Stretches) and Compressions of Functions and their Graphs
   - Reflection in the *y*-axis
   - Reflection in the *x*-axis
   - Invariant Points
   - Horizontal and Vertical Expansions and Compressions

5. Section 1.3: Combining Transformations
   - Does Order Matter?
   - Applying Multiple Transformations
   - Determining the Equation of a Transformed Graph of $f(x)$ from a Graph

6. Section 1.4: Inverses of Functions and Relations
   - Graphing Inverses
   - Writing the Equations of Inverses

→ 7. Section 3.1: Characteristics of Polynomial Functions and their Graphs (should we get this far)

Homework: This depends on how far we get today.

Readings: Section 3.1 (pages 106 to 113, Section 3.2 (pages 118 to 123), Section 3.3 (pages 126 to 133).
Practice from Textbook to try:
Section 1.1: pages 12 to 14, Practise 2, 3c,d, 4a,c, 5, 8, 11 (if you have not already tried them)
Section 1.2: pages 28 to 30, Practise 3b, 4b, 5, 6, 7, 9, 12.
Section 1.3: pages 38 to 40, Practise 4, 5a, 6, 7a, b, c, d, 8, 9c, e, 10a, b.
Section 1.4: pages 51 to 54, Practice 1b, 2a, 3a,c, 5a, e, 8b, 12a,
Hand-in Assignment: Begin working on the Chapter 1 Hand-in Assignment. It will likely be due on **Tuesday, January 25**.

Ch 1 test? Maybe Thursday, Jan 27

Last ~~Day~~:

Horizontal and vertical translations of functions

$y=f(x)$ if x is replaced by $x-h$ to get $y=f(x-h)$, the function and its graph are translated h unit to the right (→)

3 units to right

2

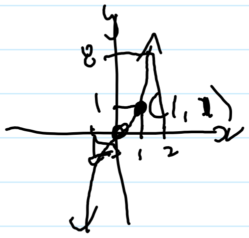
the right ($\longrightarrow$)

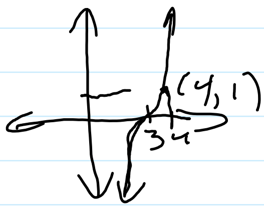
$y = x^3$ $\quad$ $y = (x-3)^3$ $\Leftarrow$ 3 units to right

$y = (x+3)^3$ $\quad$ $y = (x - (\overset{h}{-3}))^3$
shift 3 units left.

each value of x increases by 3 units

mapping notation $(x, y) \longrightarrow (x+3, y)$

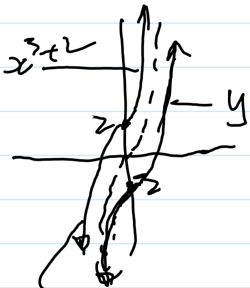




If y is replaced by $y-k$ in a function $y=f(x)$ to get $y-k=f(x)$, the function is translated k units up.

this can be written as $y=f(x)+k$

$y=x^3$ $\quad$ $y-2=x^3$ or $y = \underbrace{x^3-2}_{\text{2 units up}}$



$y=x^3+2$

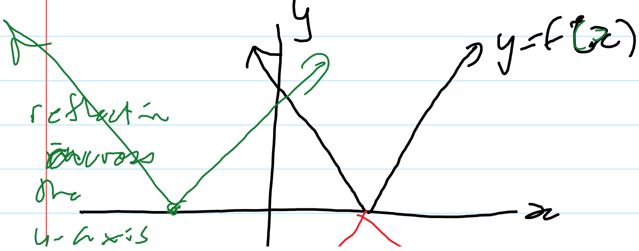
$y=x^3-2$ $\quad$ $y+2=x^3$ or $y=x^3-2$, the shift is 2 units down.

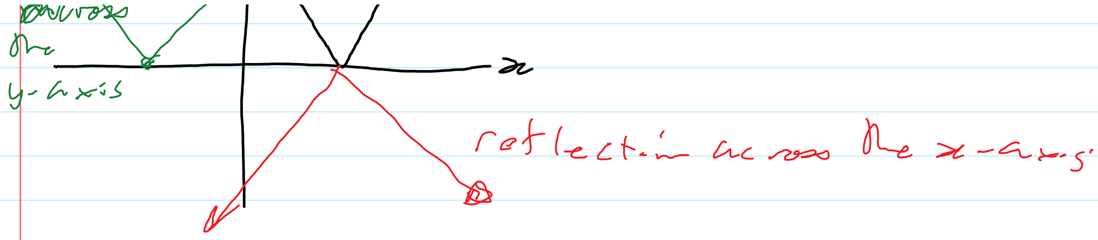
$(x, y) \longrightarrow (x, y-2)$.

$y=x^3+2$ $\quad$ mapping notation $(x, y) \rightarrow (x, y+2)$

Reflections of functions and their graphs

- functions and their graph can be reflect across both the x-axis and the y-axis.





- If y is replaced with $-y$ in a function $y=f(x)$ to get $-y=f(x)$ or $y=-f(x)$, the function is reflected across the <u>x-axis</u>.

- If x is replaced with $-x$ in a function $y=f(x)$ to get $y=f(-x)$, the function is reflected across the <u>y-axis</u>.

If a function is reflected across the x-axis, all points on the x-axis (if any) are invariant points.

If a function is reflected across the y-axis, all points on the y-axis (if any) are invariant points.

<u>Horizontal expansions and compressions</u> and <u>vertical expansions and compressions</u> "stretches"

<u>Vertical Expansion by a factor "a"</u>

If y is replaced by $\frac{1}{a}y$ (y/a) in the function $y=f(x)$ to get $\frac{1}{a}y=f(x)$ and $|a|>1$, then the function is vertically expanded by a factor of a.

Why $|a|$? ; "a" could be positive or negative.

If "a" is negative, there would also be a reflection across the x-axis.

Note that $\frac{1}{a}y = f(x)$ can be written as $y = af(x)$.

Why must $|a|$ be bigger than 1??

If "a" equals 1/2, then $y = \frac{1}{2}f(x)$ or $2y = f(x)$.

$2 = \frac{1}{\frac{1}{2}}$

If $0 < |a| < 1$, we get a vertical compression by a factor of a.

eg. $y = \frac{1}{2}f(x)$) all y-values compressed by a factor of $\frac{1}{2}$

or $2y = f(x)$

Horizontal expansion

If x is replaced by "bx" in the function $y = f(x)$, there is a horizontal expansion by a factor of $\left|\frac{1}{b}\right|$ $(|b| > 1)$

In order to have a horizontal expansion by a factor of 2, b has to equal $\frac{1}{2}$. $\frac{1}{b} = \frac{1}{\frac{1}{2}} = 2$..

If $0 < |b| < 1$ we get a horizontal expansion by a factor of $\frac{1}{b}$) $\frac{1}{b}$ would be > 1

If $|b| > 1$, we a horizontal compression by a factor of $1/b$.

eg. $y = f(x)$ if $y = f(\frac{1}{2}x)$

factor $= \frac{1}{\frac{1}{2}} = 2$ i.e. a HE by factor 2 if $y = f(3x) = f(\frac{1}{\frac{1}{3}}x)$

hor. compression by a factor of 3.

$y = f(\frac{3}{4}x)$

reciprocal $= \frac{4}{3}$ $\quad \frac{4}{3} > 1 \Rightarrow$ Hor. expansion by a factor of $\frac{4}{3}$.

All points on the y-axis have an x-coordinate of $x = 0$.

During a horizontal compression or expansion, points on the y-axis are invariant points.

During vertical compressions or expansions, all points on the x-axis are invariant. Those are points where $\underline{y = 0}$.