PC 12 Session 13

February 21, 2022 9:07 AM

Pre-Calculus 12 Session 13
Tuesday, February 22, 2022

1. Last Day's Homework:
   - Practice: Section 4.2: pages 186-188, Practise 2a), c), e), 2a), c), e), 3a), c), 4a), c), e), h), i), 5a), c), e), 6, 7, 9, 13, 18.
   - Readings: Nothing new.
   - Hand-in Assignments and other things: The Chapter 4 Hand-in Assignment ~~may possib~~ly be due on ~~Tuesday, February 22~~, but only if we finish off Chapter 4 today which is really doubtful. will be due Tues, Mar 1
2. A More about Section 4.3: Trigonometric Ratios
3. Section 4.4: Introduction to Trigonometric Equations
4. Section 5.1: Graphing Sine and Cosine Functions
5. Section 5.2: Transformations of Sinusoidal Functions

Homework: This depends on how far we get today.

Readings: Section 5.4 pages 266 to 274.

Practice from Textbook to try:

Section 4.3: pages 201-203, Practise 1a), c), e), g), i), k), 2a), c), e), g), i), k), 3a), c), e), 6a), c), e), 9a), c), e), 10 (all parts), 11 (all parts), 12a), c).

Section 4.4: pages 211-213, Practise 1a), c), 2, 3a), c), 4a), c), 5a), c), e), 6a), c), e), 7 (all), 9, 13, 16.

The Chapter 4 Review (pages 215 to 217) and the Chapter 4 Practice Test (pages 218 and 219).

Section 5.1: 1, 2, 3, 4a), d), 5b), d), 6, 7a), c), 8a), c), 9a), c), 10, 11a), c), 14, 18.

Section 5.2: Practise 2 to 7, 10, 14, 15a), c), 16a), c).

Hand-in Assignments: You should begin working on the Chapter 4 Hand-in Assignment. That assignment will possibly be due in next class. The Chapter 4 Test may possibly be on ~~Tuesday,~~ Thurs ~~March 1.~~ Mar 3

With a completed Unit Circle, you can quickly determine the exact value of the sine (y) and cosine (x) of any special angle or any angle that is coterminal with a special angle OR has $30°, 45°, 60°$ $\left(\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}\right)$ as a reference angle.

With a bit of math, you can also determine

The tangent, cotangent, secant and cosecant of those same angle.
y/x x/y $1/x$ $1/y$

3. Determine the exact value for the following questions (do not use a calculator).

a) $\sin\frac{\pi}{3} = y$
$= \frac{\sqrt{3}}{2}$

b) $\cos\frac{3\pi}{4}$ (x)
$= -\frac{\sqrt{2}}{2}$
S A
T C

e) $\cos\frac{3\pi}{2} = x = 0$

f) $\csc\frac{\pi}{4}$

$\csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\frac{\pi}{4}}$

$\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$\csc\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$\csc\frac{\pi}{4} = \frac{1}{\frac{\sqrt{2}}{2}} = 1 \times \frac{2}{\sqrt{2}}$
$= \frac{2}{\sqrt{2}}$

g) $\tan\frac{5\pi}{3}$

$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
$\theta_R = \frac{\pi}{3}$
$\frac{5\pi}{3}$
$\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$
x y

$\tan\frac{5\pi}{3} = \frac{y}{x} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2} \times \frac{2}{1} =$
$= -\sqrt{3}$

h) $\sin\left(-\frac{3\pi}{4}\right) = \boxed{\pm}\frac{\sqrt{2}}{2}$??
$= -\frac{\sqrt{2}}{2}$
S A
T C
QIII

$\frac{6\pi}{6} = \pi$

i) $\cot\frac{5\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\cot\theta = \frac{1}{\tan\theta}$

$\tan\theta = \frac{y}{x}$

$\cot\theta = \frac{x}{y}$

$\cot\frac{5\pi}{6} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}$

$= -\frac{\sqrt{3}}{2} \times \frac{2}{1} = -\sqrt{3}$

j) $\sec\frac{11\pi}{6}$

$= \frac{1}{\cos\frac{11\pi}{6}}$

$\cos\frac{11\pi}{6} = x$
$= \frac{\sqrt{3}}{2}$

$\sec\frac{11\pi}{6} = \frac{2}{\sqrt{3}}$

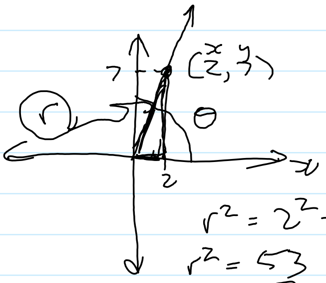
$\sec\frac{11\pi}{6} = \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{2\sqrt{3}}{3}$
✓✓ (1.1547..)

Other ways to get exact values of the trig ratios for angles in standard position.

1) You are given the coordinates of a point on the terminal arm of the angle but the point is NOT the point where the terminal arm intersects the unit circle.

eg. The point A(2,7) lies on the terminal arm of an angle θ drawn in standard position. Determine exact values of all 6 trig ratios for θ

WITHOUT Calculating θ.



$\cos\theta = \frac{x}{r} = \frac{2}{\sqrt{53}} \quad \left(= \frac{2\sqrt{53}}{53}\right)$

$\sin\theta = \frac{y}{r} = \frac{7}{\sqrt{53}} \quad \left(= \frac{7\sqrt{53}}{53}\right)$

$r^2 = 2^2 + 7^2$

$r^2 = 53$

$r = \sqrt{53}$

$\tan\theta = \frac{y}{x} = \frac{7}{2}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{\sqrt{53}}{2}$

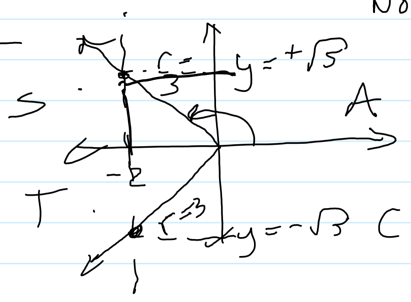
$\csc\theta = \frac{1}{\sin\theta} = \frac{r}{y} = \frac{\sqrt{53}}{7}$

$\cot\theta = \frac{1}{\tan\theta} = \frac{x}{y} = \frac{2}{7}$

Given the exact value of any <u>one</u> of the 6 trig ratios for an angle drawn in S.P., you can determine the other 5 ratios. However, there may be more than one possible answer.

eg. Given that $\cos\theta = -2/3$, determine the possible exact value(s) of all other 5 trig ratios.

$\cos\theta = \frac{x}{r} = \frac{-2}{3}$



Note: $\cos\theta < 0$ in Q II AND Q III

$r^2 = x^2 + y^2$

$3^2 = (-2)^2 + y^2$

$9 = 4 + y^2 \qquad y^2 = 5 \quad \therefore y = \pm\sqrt{5}$

In Q II: $\sin\theta = \frac{y}{r} = \frac{\sqrt{5}}{3}$ ✓

$\tan\theta = \frac{y}{x} = \frac{\sqrt{5}}{-2} = -\frac{\sqrt{5}}{2}$

$\sec\theta = -3/2$

$\csc\theta = \frac{1}{y} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$ ✓

$\cot\theta = \frac{x}{y} = \frac{-2}{\sqrt{5}}$

$\left(= \frac{-2\sqrt{5}}{5}\right)$.

In Q III: $\sin\theta = -\frac{\sqrt{5}}{3}$

$\tan\theta = \frac{-\sqrt{5}}{-2} = \frac{\sqrt{5}}{2}$ ✓

$\sec\theta = -3/2$

$\csc\theta = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$

$\cot\theta = \frac{-2}{-\sqrt{5}} = \frac{2}{\sqrt{5}}$ ✓

$\left(= \frac{2\sqrt{5}}{5}\right)$.

Solving Trigonometric Equations

Given the value of a trig ratio for an angle in SP, we can determine the value of the angle.

If we recognize that the given trig ratio is that of a "special angle", we can use the Unit Circle to determine the measure of the angle.

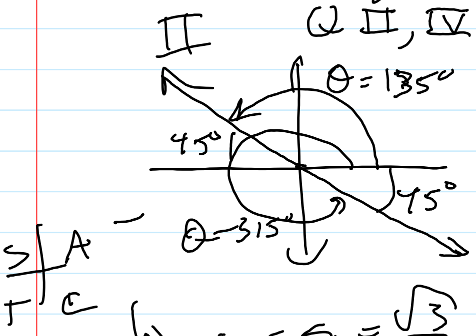
This is when the sine, cosine or tangent of the angle is $0, \pm 1, \pm \frac{\sqrt{3}}{2}, \pm \frac{\sqrt{2}}{2}, \pm \frac{1}{2}$ or is undefined
$\pm\sqrt{3}$

OR if one of the reciprocal ratios is $0, \pm 1, \pm \frac{2}{\sqrt{3}}$, $\pm \frac{2}{\sqrt{2}}, \pm 2, \pm\sqrt{3}$ etc...

eg Section 4.3 Ex.

7a) $\tan\theta = -1$, domain $0° \le \theta < 360°$

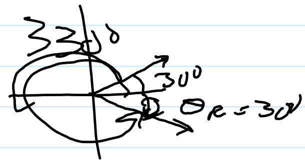
Q II, IV $\quad \theta_R = 45°$ because $\tan 45° = 1$



In Q II
$\theta = 135°$

In Q IV
$\theta = 315°$

b) $\cos\theta = \frac{\sqrt{3}}{2}$; $-180° \le \theta < 180°$

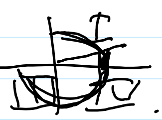


Q I, IV
$\cos\theta > 0$

In Q I, $\theta = 30°$

In Q IV, $\theta = -30°$

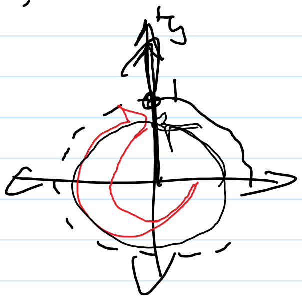
c) $\csc\theta = 2$ $\quad (-180 \le \theta < 90°)$



$\frac{1}{\sin\theta} = \frac{2}{1}$ $\quad \sin\theta = \frac{1}{2}$ (+) $\quad$ Q I, II.

In Q I, if $\sin\theta = \frac{1}{2}$, $\theta = 30°$

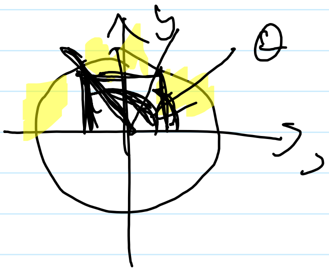
d) $\overset{(y)}{\sin}\,\theta = 1$ $\quad (-360^\circ \le \theta < 360^\circ)$



$\theta = -270^\circ, 90^\circ$

8. When restrictions on the domain are given in radians, YOU MUST GIVE YOUR ANSWER IN RADIANS

a) $\sin\theta = \frac{\sqrt{3}}{2}$ $\quad (0 \le \theta < 2\pi)$

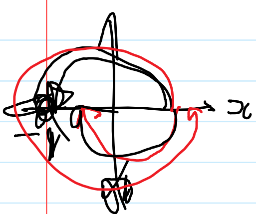


$\theta = \frac{\pi}{3}, \frac{2\pi}{3}$

b) $\sec\theta = -\frac{1}{1}$ $\quad (-\pi \le \theta < 2\pi)$

$\underset{x}{\cos}\,\theta = \frac{1}{-1} = -1$

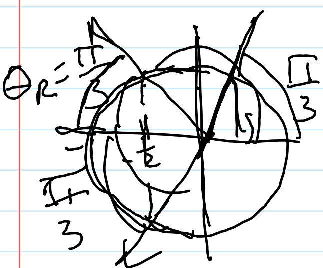
$\theta = -\pi, \pi$



in Q II, III

c) $\cos\theta = -\frac{1}{2}$

$(0 \le \theta < 2\pi)$



$\cos\frac{\pi}{3} = \frac{1}{2}$

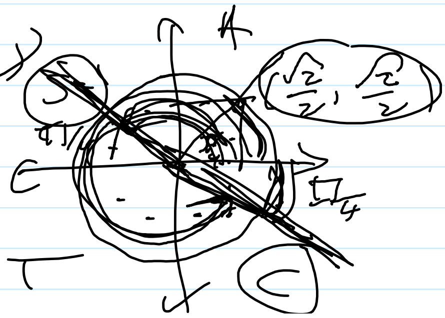
$\theta_R = \frac{\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

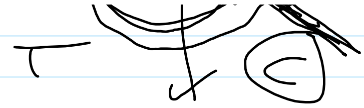
d) $\cot\theta = -1$ $\quad (-\pi \le \theta < 2\pi)$

$\frac{1}{\tan\theta} = \frac{-1}{1}$

$\tan\theta = \frac{1}{-1} = -1$ Q II or IV



$\tan\theta = \frac{1}{-1} = -1$ Q.II or IV



$\theta = -\frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$

If θ is NOT a "special angle", you may sometimes be able to use your calculator to solve for θ to get approximate answer(s).

eg Solve $\sin\theta = 0.68$ over $0 \le \theta < 360°$

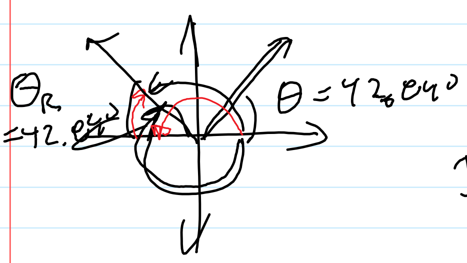
Round your answer to 2 dec. places.

Because $\sin\theta > 0$, θ is in QI or QII.

- using your calculator to get θ, gives:

$\theta = \sin^{-1}(0.68) = 42.8436...°$

$= 42.84°$ ⟸ the QI answer.



The QII angle has $\theta_R = 42.84°$

$\therefore \theta = 180° - 42.84°$

$= 137.16°$.

If the restriction on the domain includes negative angles, you must also consider those solutions.

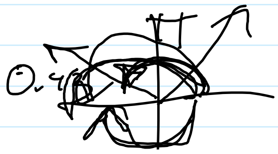
eg. $\sin\theta = 0.68$ solve over $-360° \le \theta < 360°$

$\theta = 42.84°, 137.16°$ and $-(180° + 42.84°)$

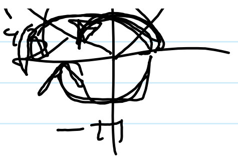
$= -222.84°$

RADS

9a). $\sin\theta = \pm 0.42$; $(-\pi \le \theta < \pi)$
-3.14 $+3.14$



4a). $\sin\theta = 0.42$; $(-\pi \le \theta < \pi)$
$-3.14 \qquad +3.14$



$\sin\theta > 0$ in QI & QII

Put your calculator in radian mode

$\theta = \sin^{-1}(0.42) = 0.43$ (QI)

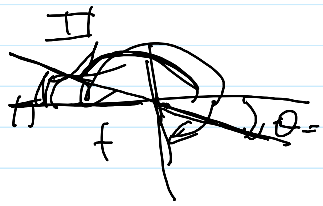
The QII angle has $\theta_R = 0.43$

$\therefore \theta = \pi - 0.4334... = \underline{\underline{2.71}}$.

b) $\cot\theta = -4.87$ (recip.) $\qquad -\frac{\pi}{2} \le \theta \le \pi$

$\tan\theta = \frac{1}{-4.87} \qquad -1.57 \le \theta \le 3.14$

$= -0.2053388...$ $\qquad$ QIV $\qquad \pi$ ~~I~~



$\theta = \tan^{-1}(-0.2053388...) = -0.20252372$

$\theta = -0.20$ (QIV).

The QII angle has a reference angle of $0.2025..$

$\theta \doteq \pi - 0.2025.. = \underline{\underline{2.94}}$.