Typo in last question!

Chapter 4 Hand-in Assignment – Trigonometry

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Unless a question says differently, round to 2 decimal places when rounding is necessary.

1. Convert each angle to degree measure.

a) $\frac{7\pi}{8} \times \frac{180^\circ}{\pi} = \frac{315}{2} = \boxed{157.5^\circ}$

b) 4.2 radians $\times \frac{180^\circ}{\pi}$

$= 240.64227\ldots$

$= \boxed{240.64^\circ}$

2. Convert each angle to radian measure, in simplest ***exact form***. (Answers will include π)

a) -200°

$\frac{-200^\circ \times \pi}{180^\circ} = \boxed{\frac{-10\pi}{9}}$

b) 1040°

$\frac{1040^\circ \times \pi}{180^\circ} = \boxed{\frac{52\pi}{9}}$

3. Convert each angle to radian measure, in ***approximate form***.

a) $258^\circ \times \frac{\pi}{180^\circ}$

$= 4.502949$

$= \boxed{4.50}$

b) -95°

$-95^\circ \times \frac{\pi}{180^\circ} = -1.65806\ldots$

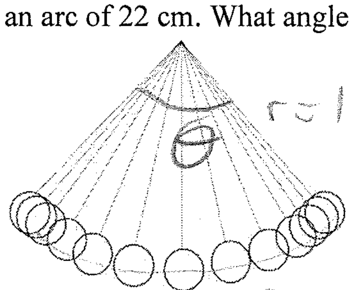
$= \boxed{-1.66}$

4. Find the arc length subtended by an angle measuring 81° in a circle with radius 18 cm.

$a = r\theta = 18\left(\frac{81^\circ \times \pi}{180^\circ}\right) = 25.4469\ldots$

$= \boxed{25.45\text{ cm}}$

5. Suppose that a clock's pendulum has a length of 15 cm, and it swings back and forth, making an arc of 22 cm. What angle does the pendulum pass through in one swing, in ***degree measure***?



$r = 15\text{ cm}$

$a = 22$

$a = r\theta$

$\theta = \frac{a}{r} = \frac{22}{15}$ rad.

$\theta = \frac{22}{15} \times \frac{180^\circ}{\pi} = 84.0338$

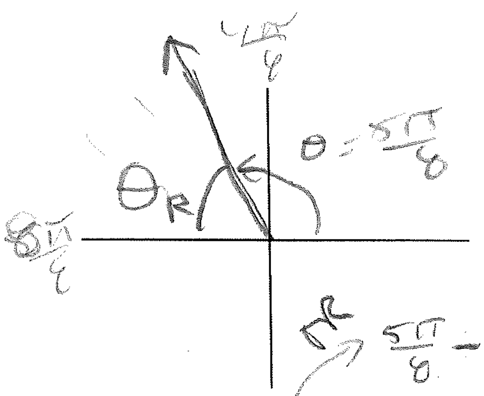
$= \boxed{84.03^\circ}$

6. For each angle below:

- graph it in standard position
- find the measure of one angle that is ***coterminal*** to the given angle
- find the ***reference angle*** to the given angle

a) $\frac{5\pi}{8}$

(Give coterminal & reference angles in exact radian measure)



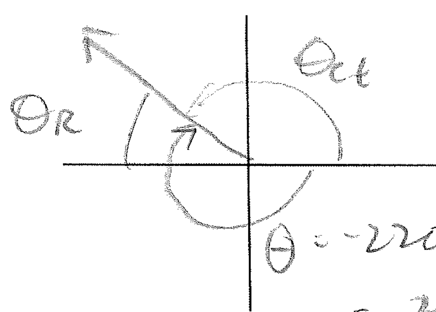
or $\frac{5\pi}{8} - \frac{16\pi}{8} = \boxed{-\frac{11\pi}{8}}$

Coterminal: $\frac{5\pi}{8} + \frac{16\pi}{8} = \boxed{\frac{21\pi}{8}}$

Reference: $\theta_R = \boxed{\frac{3\pi}{8}}$

b) -220°

(Give coterminal & reference angles in degree measure)



or $-220^\circ - 360^\circ = \boxed{-580^\circ}$

Coterminal: $-220^\circ + 360^\circ = \boxed{140^\circ}$

Reference: $\boxed{40^\circ}$

7. Find the x-coordinate of all points on the unit circle that have a y-coordinate of $\frac{2}{5}$.

Give answers in fractional form, not decimal form.

$x^2 + y^2 = 1$

$x^2 = 1 - y^2$

$x^2 = 1 - \left(\frac{2}{5}\right)^2$

$x^2 = \frac{25}{25} - \frac{4}{25} = \frac{21}{25}$

$x = \pm\sqrt{\frac{21}{25}} = \boxed{\frac{\pm\sqrt{21}}{5}}$

8. Find each value, correct to **<u>three decimal places</u>**. (Use a calculator!)

a) $\csc 185^\circ$

$= \frac{1}{\sin 185^\circ}$

$= \boxed{-11.474}$

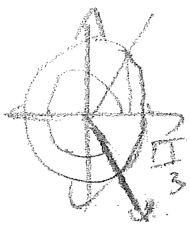
b) $\cot\left(\frac{3\pi}{7}\right)$

$= \frac{1}{\tan\left(\frac{3\pi}{7}\right)}$

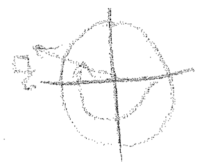
$= \boxed{0.228}$

9. Find the EXACT (x, y) coordinates where the terminal arm of each angle listed below intersects the unit circle:

a) $\frac{5\pi}{3}$ $\quad\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$



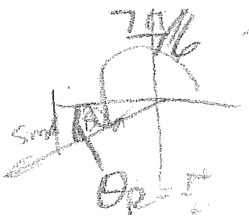
b) $-\frac{7\pi}{6}$ $\quad\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$



c) $-\frac{3\pi}{4}$ $\quad\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

10. Find the angle measure, in BOTH radians and degrees, that corresponds with each point on the unit circle:

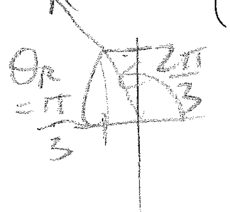
a) $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$



$\theta_R = \frac{\pi}{6}$

$\boxed{\frac{7\pi}{6}}$ $\boxed{210^\circ}$

b) $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



$\theta_R = \frac{\pi}{3}$ $\quad \frac{2\pi}{3}$

$\boxed{\frac{2\pi}{3}}$ $\boxed{120^\circ}$

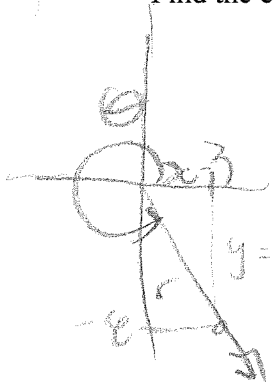
c) $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\theta = \frac{7\pi}{4}$

$\theta_R = \frac{\pi}{4}$

$\boxed{\frac{7\pi}{4}}$ $\boxed{315^\circ}$

11. Suppose the terminal arm of a standard position angle θ passes through the point $(3,-8)$. Find the exact value of all six trigonometric ratios for angle θ, in fractional form.



$r^2 = 3^2 + (-8)^2$

$r^2 = 73$

$r = \sqrt{73}$

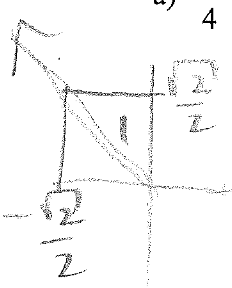
$\cos\theta = \frac{x}{r} = \frac{3}{\sqrt{73}}$ $\qquad \sec\theta = \frac{\sqrt{73}}{3}$

$\sin\theta = \frac{y}{r} = -\frac{8}{\sqrt{73}}$ $\qquad \csc\theta = -\frac{\sqrt{73}}{8}$

$\tan\theta = \frac{y}{x} = -\frac{8}{3}$ $\qquad \cot\theta = -\frac{3}{8}$

12. Find the ***exact value*** of all six trigonometric ratios for each angle θ. Give answers in simple form (no complex fractions).

a) $\frac{3\pi}{4}$



$\sin\theta = \boxed{\frac{\sqrt{2}}{2}}$ $\quad \cos\theta = \boxed{-\frac{\sqrt{2}}{2}}$ $\quad \tan\theta = \frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{-1}$

$\csc\theta = \boxed{\frac{2}{\sqrt{2}}}$ $\quad \sec\theta = \boxed{-\frac{2}{\sqrt{2}}}$ $\quad \cot\theta = \boxed{-1}$

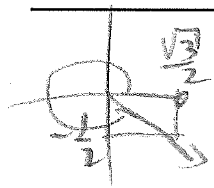
$\boxed{\text{or} = \sqrt{2}}$ $\quad$ or $= \boxed{-\sqrt{2}}$

$\left(\frac{2}{\sqrt{2}} = \frac{\sqrt{2} \times \sqrt{2}}{\sqrt{2}} = \sqrt{2}\right)$

$\sin\theta = \boxed{0}$ $\cos\theta = \boxed{-1}$ $\tan\theta = \frac{0}{-1} = \boxed{0}$

b) $-\pi$

$\csc\theta = \frac{1}{0} \boxed{= \text{undef.}}$ $\sec\theta = \frac{1}{-1} \boxed{= -1}$ $\cot\theta = \frac{-1}{0} = \boxed{\text{undef.}}$



c) 330°

$\sin\theta = \frac{-1}{2}$ $\cos\theta = \frac{\sqrt{3}}{2}$ $\tan\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{-1}{\sqrt{3}} \left(\text{or } \frac{-\sqrt{3}}{3}\right)$

$\csc\theta = -2$ $\sec\theta = \frac{2}{\sqrt{3}} \left(\text{or } \frac{2\sqrt{3}}{3}\right)$ $\cot\theta = -\frac{3}{\sqrt{3}} \boxed{= -\sqrt{3}}$

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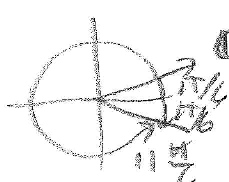
14. Solve these trigonometric equations algebraically.
- Give answers in EXACT form when possible.
- If domain is in radians, give answers in radian measure

a) $\cos\theta = \frac{\sqrt{3}}{2}$, $\underline{0 \le \theta < 2\pi}$

$\cos\theta > 0 \therefore$ Q I & IV

$\theta_R = \frac{\pi}{6}$

$\boxed{\theta = \frac{\pi}{6}, \frac{11\pi}{6}}$



QI, $\theta = \boxed{\frac{\pi}{6}}$

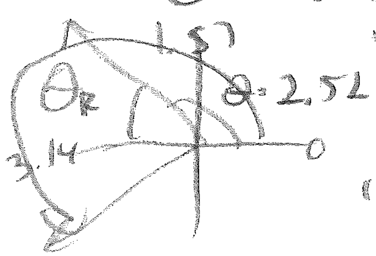
QIV, $\theta = 2\pi - \frac{\pi}{6} = \boxed{\frac{11\pi}{6}}$

b) $\cos\theta = -0.813$, for $0 \le \theta < 2\pi$

Q II, III

QII

$\theta = \cos^{-1}(0.813) = 2.52008 = \boxed{2.52}$



$\theta_R = \pi - 2.52008$

$= 0.62151\ldots$

in QIII,

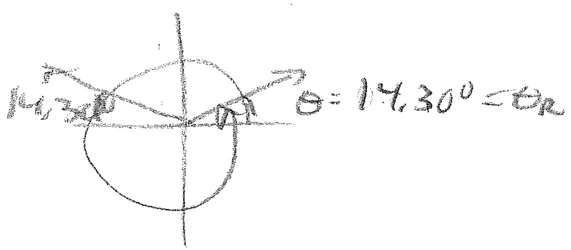
$\theta = \pi + 0.62151\ldots = 3.76310 = \boxed{3.76}$

c) $\sin\theta = 0.247$, for $0^\circ \le \theta \le 720^\circ$

QI, II

QI

$\theta = \sin^{-1}(0.247) = 14.300050 \approx \boxed{14.30^\circ}$



in QII,

$\theta = 180^\circ - 14.300050$

$= 165.6999...$

$= \boxed{165.70^\circ}$

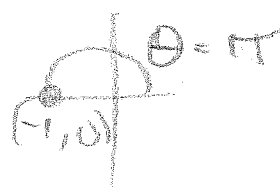
and $\theta = 14.30^\circ + 360^\circ = \boxed{374.30^\circ}$ (QI)

and $\theta = 165.70^\circ + 360^\circ = \boxed{525.70^\circ}$ (QII)

d) $2\cos\theta + 1 = -1$, $0 \le \theta < 2\pi$

$2\cos\theta = -1 - 1 = -2$

$\cos\theta = -1 \Rightarrow$ quadrantal angle.



$\boxed{\theta = \pi}$

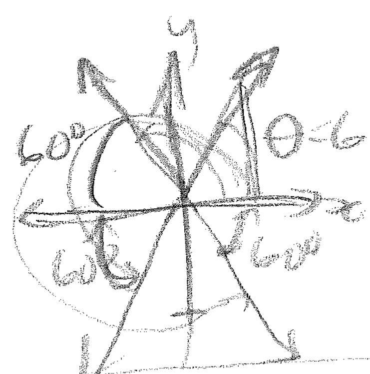
e) $4\sin^2\theta - 3 = 0$, $0^\circ \le \theta < 360^\circ$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \frac{\pm\sqrt{3}}{2}$

QI, II, QIII, IV

+ all Quadrants



$\theta = 60^\circ = \theta_R$

QI $\theta = 60^\circ$

QII:
$\theta = 180^\circ - 60^\circ$
$= 120^\circ$

QIII:
$\theta = 180^\circ + 60^\circ$
$= 240^\circ$

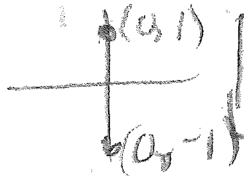
$\boxed{\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ}$

QIV, $\theta = 360^\circ - 60^\circ$
$= 300^\circ$.

f) $\sqrt{2}\cos^2\theta - \cos\theta = 0$, $0 \le \theta < 2\pi$

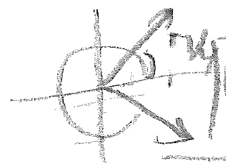
$\cos\theta(\sqrt{2}\cos\theta - 1) = 0$

$\cos\theta = 0 \Rightarrow$ $\qquad$ $\sqrt{2}\cos\theta - 1 = 0$



$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\cos\theta = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$



$\boxed{\theta = \frac{\pi}{4}, \frac{7\pi}{4}}$

$$\boxed{\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{7\pi}{4}}$$

g) $2\tan^2\theta - 7\tan\theta - 3 = 0$, $0^\circ \le \theta < 720^\circ$

$2x + 3 = +6$

$\square \times \square = +6$

$\square + \square = -7$

$-1 \quad + -6$

$2\tan^2\theta - 7\tan\theta + 3 = 0$

$2\tan^2\theta - 1\tan\theta - 6\tan\theta + 3 = 0$

$\tan\theta(2\tan\theta - 1) - 3(2\tan\theta - 1) = 0$

$(\tan\theta - 3) = 0 \qquad (2\tan\theta - 1) = 0$

$\tan\theta = 3 \qquad \tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}(3)$

$= 71.56505118^\circ$

in QIII $\quad = \underline{71.57^\circ}$

QI

$\theta = \tan^{-1}(\frac{1}{2}) = 26.56505118^\circ = \underline{26.57^\circ}$

in QIII, $\theta = 180^\circ + 26.565...^\circ$

$= 206.565...^\circ$

$= \underline{206.57^\circ}$

$\theta = 180^\circ + 71.565...$

$= 251.565^\circ$

$= \underline{251.57^\circ}$

OVER $\longrightarrow$

9) Note domain: $0 \le \theta < 720^\circ$

In Q I

$\theta = \boxed{26.57^\circ}, \boxed{71.57^\circ}$ AND

$\theta = 26.57 + 360^\circ = \boxed{386.57^\circ}$

AND $\theta = 71.57^\circ + 360^\circ = \boxed{431.57^\circ}$

In Q III

$\theta = \boxed{206.57^\circ}, \boxed{251.57^\circ}$ AND

$\theta = 206.57^\circ + 360^\circ = \boxed{566.57^\circ}$

AND $\theta = 251.57^\circ + 360^\circ = \boxed{611.57^\circ}$