Chapter 8 Note Package

Wednesday, April 6, 2022 12:53 PM

? exponent

$3^{4} = 81$ argument.

base

$3^{\boxed{?}} = 729$

Chapter 8: Logarithmic Functions

8.1 Understanding Logarithms

A logarithm tells how many copies of one number we need to multiply together, to create a different number.

For example:
How many 4's do we have to multiply together to get 64?
$4 \times 4 \times 4 = 64$, which shows we have to multiply 3 of the "4's" to produce 64
This tells us the ***logarithm*** is 3.

$4^3 = 64$

$\log_4 64 = 3$

Because $4 \times 4 \times 4 = 64$, we can say: $\log_4(64) = 3$

- we can read this as "The logarithm base 4 of 64 is equal to 3"
- we can shorten it a bit, and say "log base 4 of 64 equals 3"

base 4 log

$$\log_4(64) = 3$$

argument; base; exponent or log; argument.

$\text{base}^{\text{power}} = \text{argument}$

A logarithm tells us how many copies of the BASE we need to multiply together, to create the ARGUMENT – in other words, the logarithm is the ***exponent*** we raise the base to, in order to produce the argument.

Try These

1. $\log_4(16) = 2$ $4^{2} = 16$
2. $\log_3(27) = 3$ $3^3 = 27$ $\log_3(3^3) = 3$
3. $\log_{0.5}(0.5) = 1$
4. $\log_a(a^7) = 7$
5. $\log_2(2^{-3}) = -3$
6. $\log_1 7 = ?$ no solution $1^{?} = 7$
7. $\log_5\left(\frac{1}{25}\right) = -2$ $\frac{1}{25} = \frac{1}{5^2} = 5^{-2}$ $\log_5(5^{-2}) = -2$
8. $\log_3 0 = ?$ no solution. $3^{?} = 0$
9. $\log_2(-4) =$ no solution. $2^{?} = -4$ $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$
10. $\log_6\left(\sqrt[2]{6}\right) =$ $\log_6(6^{\frac{1}{2}}) = \frac{1}{2}$
11. $\log_2\left(\sqrt[3]{2^4}\right) = 4/3$ $\log_2(2^{4/3})$
12. $\log_8(1) = \log_8(8^0) = 0$ $8^{0} = 1$

For a logarithm to make sense, we need the argument and the base to obey these restrictions:

argument > 0 (positive).

base $\neq 1$, base > 0 (base is positive)

$\log 100 = \log_{10} 100 = \log_{10}(10^2) = 2$

no base means it's a base-10 (or common) logarithm

Notation

log base 10 is called **COMMON** log
$\log_{10} x$ *is written* $\log x$

log base *e* is called **NATURAL log**
$\log_e x$ *is written* $\ln x$

Notation

log base 10 is called **COMMON log** — $\log_{10} x$ *is written* $\log x$

log base *e* is called **NATURAL log** — $\log_e x$ *is written* $\ln x$

The number *e* is a very important irrational number. Its decimal expansion starts out:

$e \approx$ **2.71828182845904523536028747135...**

https://www.popularmechanics.com/science/math/a24383/mathematical-constant-e/

Changing Form

Exponents and logarithms are closely connected. Look at these two equations:

$\log_4 64 = 3$ and $4^3 = 64$

logarithmic form — exponential form

base, power, exponent, argument

power = log value

base = argument.

Both equations show the relationship between the numbers 4, 3 and 64. We need to know how to change equations from one form to the other, as in some questions one form is better than the other.

To Try

1. Change form.

a) $\log_6 216 = 3$ — $6^3 = 216$

b) $\log_p q = r$ — $p^r = q$

c) $\log 1000 = 3$ — $\log_{10}$ — $10^3 = 1000$

d) $7^2 = 49$ — $\log_7 49 = 2$

e) $5^{x+y} = a$ — $\log_5 a = x+y$

f) $49^{1/2} = 7$ — $\log_{49} 7 = \frac{1}{2}$

2. Solve for *x*.

$3^{2x} = 9^{3x+1}$

$3^{2x} = (3^2)^{3x+1}$

$2x = 6x+2$

a) $\log_2(x-1) = 3$

$2^3 = x-1$

$8 = x-1$

$+1 \quad +1$

$\boxed{x=9}$

b) $\log_6 x = -2$

$6^{-2} = x \qquad x = \frac{1}{6^2} = \frac{1}{36}$

c) $\log_x 8 = 3$

$x^3 = 8$

$x^3 = 2^3$

$x = 2$

d) $\ln x = 2$ — ln

$e^2 = x$

$x \doteq 7.389056099$

e) $\log_2(\log_9 x) = -1$

$2^{-1} = \log_9 x$

$\frac{1}{2} = \log_9 x$

$\sqrt[2]{9^1} = 9^{\frac{1}{2}} = x$

$x = 3$

f) $4^{\log_4 x} = 7$

$\log_4 x = \log_4 7$

$\boxed{x = 7}$

$4^{\log_4 7} = 7$

$4^3 = 64$

$\log_4 64 = 3$ ← exponent

base, argument

$$\boxed{a^{\log_a b} = b}$$

This is one of the laws of Logarithms!

Graphing an Exponential Function and its Inverse

a) Fill in the table below, and sketch the graph of the exponential function $\boxed{y = 2^x}$.

x	y
−2	1/4
−1	1/2
0	1
1	2
2	4

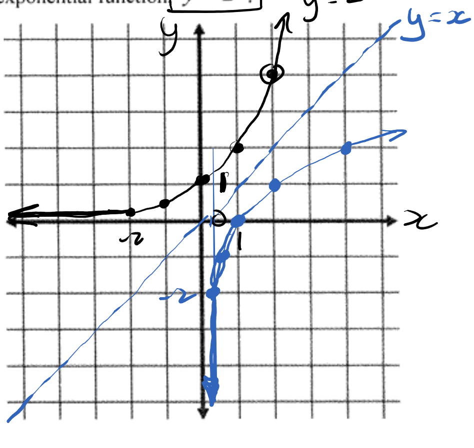
$y = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}$

$y = 2^{-1} = \frac{1}{2}$

$y = 2^0$

$y = 2^1$

$y = 2^2$



b) Identify the following:

domain $\{x \mid x \in \mathbb{R}\}$

range $\{y \mid y > 0, y \in \mathbb{R}\}$

asymptote equation $y = 0$

x-intercept, if it exists none.

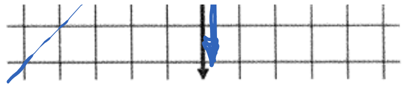
range $\{y \mid y > 0, y \in \mathbb{R}\}$

asymptote equation $y = 0$

x-intercept, if it exists none.

y-intercept, if it exists $(0, 1)$

at $y = 1$



c) Give the equation of the *inverse* of: $\boxed{y = 2^x}$ Inverse's equation is: $x = 2^y$

d) For the equation of the inverse that you found in part c), complete the table at right and sketch the graph on the grid above.

x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

e) For the inverse graph, what are its:

domain $D: \{x \mid x > 0, x \in \mathbb{R}\}$

range $R: \{y \mid y \in \mathbb{R}\}$

asymptote equation $x = 0$

x-intercept, if it exists at $x = 1$ ∴ $(1, 0)$

y-intercept, if it exists none

$x = 2^y$ $\log_2 x = y$ $y = \log_2 x$

$y = 2^x$ $y = \log_2 x$ inverses.

f) Rewrite the inverse equation from part c) in logarithmic form:

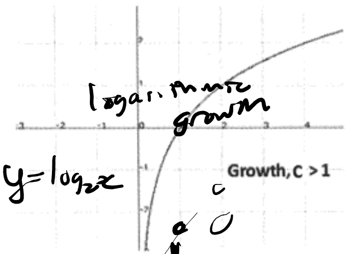
Conclusions:

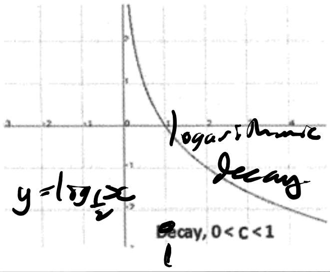
$\log_a\left(a^x\right) =$ $a^{\log_a x} =$

8.2 Transformations of Logarithmic Functions

The graphs of logarithmic functions can be grouped into two categories:

- if the logarithm's base is larger than one, the graph is ***increasing***
- if the logarithm's base is between zero and one, the graph is ***decreasing***





$y = c^x$
$c > 1$
exponential growth

$y = \frac{1}{3}^x$
$0 < c < 1$
exponential decay.

What characteristics are the ***same*** for all untransformed logarithmic graphs?

$D: \{x \mid x > 0, x \in \mathbb{R}\}$ x-int: $(1, 0)$

$R: \{y \mid y \in \mathbb{R}\}$ no y intercept

asymptote: $x = 0$ (the y-axis)

Predict what will happen to the graph of $y = \log_3 x$ when each of the following changes is made to the equation:

$y = \log_3 x - 5$ down 5

$y = -4\log_3 x$ reflect over x-axis, VE by 4

$y = \log_3\left(-\frac{2}{5}(x+3)\right)$

$y = f(x)$

$y = f(x) - 5$

$y = 1$

reflect in over y-axis HE by $5/2$, 3 units to the left.

Horizontal stretch by a factor of $\frac{1}{b}$

Horizontal translation

Vertical stretch by a factor of a

Vertical translation

$y = \log_c x$

Horizontal stretch by a factor of $\frac{1}{b}$

Horizontal translation

Vertical stretch by a factor of a

Vertical translation

$y = \log_c x$

$$y = a\log_c b(x-h)+k$$

If b < 0 then there is a reflection over the y-axis (horizontal reflection)
If a < 0 then there is a reflection over the x-axis (vertical reflection)

If the graph of a log function is transformed, the D, the R, the x-axis intercept, the y-axis intercept & the equation of the asymptote may all change.

8.3 Laws of Logarithms

Product Law:	$\log_c(MN) = \log_c M + \log_c N$
Quotient Law:	$\log_c\left(\frac{M}{N}\right) = \log_c M - \log_c N$
Power Law:	$\log_c\left(M^P\right) = P\log_c M$

$3^3 = 27$

To Try:

1. Evaluate without using use the "log" button: $\log_3 54 - \log_3 2 = \log_3\left(\frac{54}{2}\right)$
$= \log_3(27) = 3$

2. Find the value of each of the following without using a calculator:
a) $\ln 1 = 0$ b $\ln e$ $\log_e e = 1$ c) $\ln(e^4) = 4\ln e = 4\log_e e = 4(1)$
$= 4.$

3. Evaluate without using the "log" button: $\log_{14} 4 + \log_{14} 49 = \log_{14}(4 \times 49)$
$= \log_{14}(196) = \log_{14}(14^2)$
$= 2\log_{14} 14$
$= 2(1) = 2.$

Change of Base Formula: $\log_C A = \frac{\log A}{\log C}$

$\log_5 30$ $\frac{\log 30}{\log 5}$

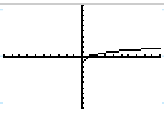
1. Evaluate. Give answer correct to 4 decimal places. $\log_2 18 = \frac{\log 18}{\log 2} = 4.169925$

2. Express as a single logarithm. $\frac{\log 30}{\log 5} = \log_5 30$

3. Rewrite this equation so you can graph it on a graphing calculator: $y = \log_4 x = \frac{\log x}{\log 4}$ ✓✓

x	y
$\frac{1}{4^2} = \frac{1}{16}$	-2
$\frac{1}{4}$	-1
1	0

$\frac{1}{4}$	-1
1	0
4	1
16	2



$y = \frac{\log x}{\log 4}$

$\log_c (ab)$
$= \log_c a + \log_c b$

1. $\log_3(4y^2) = \log_3(4 \times y^2)$ $= \log_3 4 + \log_3 y^2$ $= \log_3 4 + 2\log_3 y$	6. $\frac{1}{2}(\log b - \log c)$ $\frac{1}{2}\log b - \frac{1}{2}\log c$ $\log(b^{\frac{1}{2}}) - \log(c^{\frac{1}{2}})$ $\log\left(\frac{b^{\frac{1}{2}}}{c^{\frac{1}{2}}}\right) = \log\left(\frac{\sqrt{b}}{\sqrt{c}}\right)$ $= \log\left(\sqrt{\frac{b}{c}}\right)$
2. $2\log_4 b + 3\log_4 c$ $\log_4(b^2) + \log_4(c^3)$ $\log_4(b^2c^3)$	7. $\log\left(\frac{\sqrt{a}}{c^2}\right)$ $\log\sqrt{a} - \log(c^2)$ $\log(a^{\frac{1}{2}}) - \log(c^2)$ $= \frac{1}{2}\log a - 2\log c$
3. $\ln(ab)$ $\ln a + \ln b$	8. $\log(x^2y)^4 = \log(x^8y^4)$ $= \log x^8 + \log y^4$ $= 8\log x + 4\log y$
4. $\log\left(\frac{a}{b}\right) = \log a - \log b$	9. $3\log x - \log w^2$ $\log x^2 - \log w^2$ $\log\left(\frac{x^2}{w^2}\right)$
5. $\frac{1}{2}\log a + 2\log c$ $\log(a^{\frac{1}{2}}) + \log(c^2)$ $\log(a^{\frac{1}{2}} \cdot c^2)$ $\log(\sqrt{a} \times c^2) = \log(c^2\sqrt{a})$	10. $\log\left(\frac{1000a^2}{c}\right)$ $\log(1000a^2) - \log c$ $\log 1000 + 2\log a - \log c$ $3 + 2\log a - \log c$

$\frac{\sqrt{11}}{\sqrt{3}} = \sqrt{\frac{11}{3}}$

$10^{\square} = 1000$

More next day.

$5^{1} = 5$ $\log_c(abd) = \log_c a + \log_c b + \log_c d$

expand fully

11. $\log_5(5x\sqrt{y})$ $\log_5 5 + \log_5 x + \log_5 \sqrt{y}$ $1 + \log_5 x + \log_5 y^{\frac{1}{2}}$ $1 + \log_5 x + \frac{1}{2}\log_5 y$	16. $\log_7 y - 2\log_7 w + \log_7(5x)$ $\log_7\left[\frac{y(5x)}{w^2}\right]$
12. $\log\left(\frac{\sqrt{bc}}{a}\right)$ $\log\sqrt{bc} - \log a$ $\log(bc)^{\frac{1}{2}} - \log a$ $\frac{1}{2}\log(bc) - \log a$ $\frac{1}{2}[\log b + \log c]$ $\frac{1}{2}\log b + \frac{1}{2}\log c - \log a$	17. $\log a + 3\log b - 2\log c$ $\log\left(\frac{ab}{c^2}\right)$
13. $2\log a - 4\log b$ $\log a^2 - \log b^4$ $\log\left(\frac{a^2}{b^4}\right)$	18. $5\log_4 2 - \frac{1}{3}\log_4 8$ $\log_4 2^5 - \log_4 8^{\frac{1}{3}}$ $\log_4\left(\frac{32}{\sqrt[3]{8}}\right) = \log_4\left(\frac{32}{2}\right)$ $= \log_4 16 = \boxed{2}$ $2^5 = 32$ $= 16 \times 2$
14. $\log(a^2c)$ $\log a^2 + \log c$ $2\log a + \log c$	19. $\frac{\log_5 x}{4} - \log_5(3x)$ $\frac{1}{4}\log_5 x - \log_5 3x$ $\log_5\left(\frac{x^{\frac{1}{4}}}{3x}\right) = \log_5\left(\frac{\sqrt[4]{x}}{3x}\right)$
15. $\log\left(\frac{x}{yw}\right)$ $\log x - \log(yw)$ $\log x - [\log y + \log w]$ $\log x - \log y - \log w$	20. $2\log c - (3\log a + \log b)$ $\log\left(\frac{c^2}{a^3 b}\right)$

8.4 Logarithmic and Exponential Equations

Solving Logarithmic Equations

1. Use logarithm laws to simplify equation into one of two forms:
   - **$\log_c$ (argument) = number** $c^{\text{number}} = \text{argument}$
     - in this case, change to exponential form and solve
   - **$\log_c$ (argument) = $\log_c$ (another argument)**
     - in this case, set the two arguments equal

2. Use algebra to solve the equation you created in step 1.

3. Substitute each solution into the original equation. If the solution makes the argument become zero or a negative number, then it is an ***extraneous solution*** and must be rejected.
Invalid

$\log_c(ab)$
$\log_c a + \log_c b$

To Try:
Solve for x. Reject any extraneous solutions.

1. $\log_9 5 + \log_9 x = \log_9 30$

$\log_9(5x) = \log_9 30$
$5x = 30$
$x = 30/5 = \boxed{6}$ valid

2. $\ln x + \ln 5 = 2$

$\ln(5x) = \ln e^2$
$5x = 2$
$x = 2/5$

OR
Change to base 10.

3. $\ln 512 - \ln 8 = 3\ln x$

$\ln\left(\frac{512}{8}\right) = \ln(x^3)$
$\sqrt[3]{\frac{512}{8}} = \sqrt[3]{x^3}$
$x = \frac{\sqrt[3]{512}}{\sqrt[3]{8}} = \frac{8}{2} = 4$

4. $\log_2(x-6) = 3 - \log_2(x-4)$

See below

5. $2\log_4(x+4) - \log_4(x+12) = 1$

6. $\log_{12}(3-x) + \log_{12}(2-x) = 1$

$\ln 5x = \frac{\log 5x}{\log e}$

$\ln x + \ln 5 = 2$
$\ln(5x) = \ln 2$

$\ln 2 = \frac{\log 2}{\log e}$

$\frac{\log(5x)}{\cancel{\log e}} = \frac{\log 2}{\cancel{\log e}}$

$5x = 2$

$2^3 = 8$

$x = 2/5$

$x - 6 > 0$ $x > 6$. $x - 4 > 0$

4. $\log_2(x-6) = 3 - \log_2(x-4)$ every term must

$x-6>0$ $x>6$. $x-4>0$

4. $\log_2(x-6) = 3 - \log_2(x-4)$ every term must be written as a base 2 log.

$\log_2(2-6) = 3 - \log_2(2-4)$

$\log_2(x-6) = \log_2 8 - \log_2(x-4)$

$\cancel{\log_2}(x-6) = \cancel{\log_2}\left(\frac{8}{x-4}\right)$

$(x-4)\,x-6 = \left(\frac{8}{\cancel{x-4}}\right)\cancel{(x-4)}$

$(x-4)(x-6) = 8$

$x^2 - 10x + 24 = 8$
$\quad\quad\quad -8 \quad -8$

$1x^2 - 10x + 16 = 0$

$x^2 - 8x - 2x + 16 = 0$

$x(x-8) - 2(x-8) = 0$

$1 \times 16 = 16$

$\square \times \square = 16 = -8 \times -2$

$\square + \square = -10$

$(x-2)(x-8) = 0$

$x-2=0 \quad x-8=0$

$x = \cancel{2}$ reject. $\quad x = 8$ ✓ valid.

5. $2\log_4(x+4) - \log_4(x+12) = 1$ write as a base 4 log!

$1 = \log_4 4 \quad 4^1 = 4$

$\log_4\left[\frac{(x+4)^2}{(x+12)}\right] = \log_4 4$

$\cancel{(x+12)}\frac{(x+4)^2}{\cancel{x+12}} = 4(x+12)$

expand! $\quad (x+4)^2 \neq x^2 + 4^2$

$(x+4)^2 = 4x + 48$

$(x+4)(x+4) = 4x + 48$

$x^2 + 8x + 16 = 4x + 48$

$1x^2 + 4x - 32 = 0$ Solve for x

$(x+8)(x-4) = 0$

$x+8=0 \quad x-4=0$

$x = \cancel{-8}$ reject $\quad x = 4$ ✓ valid.

$2\log_4(x+4) - \log_4(x+12)$
$-8+4=-4$
$x > -4 \quad\quad x > -12$

6. $\log_{12}(3-x) + \log_{12}(2-x) = 1 \quad \log_{12} 12^1 = 1$

$3-1=4 \quad\quad 2-1=3$

$\log_{12}(3-x) + \log_{12}(2-x) = \log_{12} 12$

$\cancel{\log_{12}}[(3-x)(2-x)] = \cancel{\log_{12}} 12$

$6 - 3x - 2x + x^2 = 12$

$x^2 - 5x + 6 = 12$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$x-6=0 \quad x+1=0$

$x = \cancel{6}$ reject $\quad x = -1$ valid.

Solving Exponential Equations with Different Bases

In chapter 7 we solved exponential equations by making each side of the equation use the same base. When that is hard to do, we can instead solve by taking the logarithm of each side of the original equation and solving the resulting equation.

use base-10 logs or base-e logs

To Try:
Solve for *x*.

1. $3^x = 2800$

$$\frac{\log 2800}{\log 3} \neq \log\left(\frac{2800}{3}\right)$$

$$\log(3^x) = \log(2800)$$

$$\frac{x \log 3}{\log 3} = \frac{\log 2800}{\log 3}$$

$$x = \boxed{\frac{\log 2800}{\log 3}} \doteq 7.22$$

exact — approximate

2. $e^x = 2$

$$\ln(e^x) = \ln 2$$
$$x = \ln 2$$
$$\doteq 0.69$$

$$\log(e^x) = \log 2$$
$$x \log e = \log 2$$
$$x = \frac{\log 2}{\log e} \doteq 0.69$$

3. $3(4^{2x+3}) = 8^{4x-2}$

$$\log[3(4^{2x+3})] = \log[8^{(4x-2)}]$$
$$\log 3 + \log(4^{2x+3}) = \log[8^{4x-2}]$$
$$\log 3 + (2x+3)(\log 4) = (4x-2)(\log 8)$$
$$\log 3 + 2x\log 4 + 3\log 4 = 4x\log 8 - 2\log 8$$
$$+2\log 8 \quad -2x\log 4 \qquad -2x\log 4 \quad +2\log 8$$
$$\log 3 + 2\log 8 + 3\log 4 = 4x\log 8 - 2x\log 4$$
$$\frac{\log 3 + 2\log 8 + 3\log 4}{(4\log 8 - 2\log 4)} = \frac{x(4\log 8 - 2\log 4)}{(4\log 8 - 2\log 4)}$$
$$x = \frac{(\log 3 + 2\log 8 + 3\log 4)}{(4\log 8 - 2\log 4)} \doteq 1.69812\ldots$$
$$\doteq 1.70$$

$$3 = \frac{8^{4x-2}}{4^{2x+3}}$$
$$3 = \frac{(2^3)^{(4x-2)}}{(2^2)^{(2x+3)}}$$
$$3 = \frac{2^{12x-6}}{2^{4x+6}}$$
$$3 = 2^{(12x-6)-(4x+6)}$$
$$3 = 2^{8x-12}$$

$$3 = 2^{8x-12}$$
$$\log 3 = (8x-12)\log 2$$
$$\log 3 = 8x\log 2 - 12\log 2$$
$$\frac{\log 3 + 12\log 2}{8\log 2} = \frac{8x\log 2}{8\log 2}$$
$$x = \frac{\log 3 + 12\log 2}{8\log 2}$$
$$\doteq 1.69812\ldots \doteq \boxed{1.70}$$

Earthquakes, Sound, pH

Logarithms can be used to solve applications comparing the intensity of earthquakes, the intensity of sounds, and the acidity or alkalinity of solutions. The Richter scale for earthquakes, the decibel scale for sounds and the pH scale for solutions are all base 10.

$I = I_0(10)^{R-r}$ where:

I = intensity of a stronger earthquake
I_o = intensity of weaker earthquake

R = Richter magnitude of stronger earthquake
r = Richter magnitude of weaker earthquake

$\frac{I}{I_0} = 10^{R-r}$

$I = I_0(10)^{(D-d)/10}$ where:

I = intensity of a louder sound
I_o = intensity of a softer sound

D = decibel level of louder sound
d = decibel level of softer sound

$\frac{I}{I_0} = 10^{\frac{D-d}{10}}$

$I = I_0(10)^{P-p}$ where:

I = new solution that is compared to original one
I_o = "original" solution

P = larger pH reading
p = smaller pH reading

$I_2 = [H^+]$ in solution 2
$I_1 = [H^+]$ in solution 1

The # of times more acidic solution 2 is than solution 1

- A neutral solution has a pH of 7.
- Solutions with pH larger than 7 are basic, or alkaline.
- Solutions with pH smaller than 7 are acidic.

Neutral
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14
Increasing acidity — Increasing alkalinity

$pH = -\log[H^+]$ where $[H^+]$ is the hydrogen ion concentration in moles per liter

$\frac{I_2}{I_1} = \frac{(\log[H^+]_2)}{(\log[H^+])} = \frac{-pH_2}{-pH_1}$

$\frac{I_2}{I_1} = \frac{pH_1}{pH_2}$

The exponent is always a difference: larger reading – smaller reading
For sound questions, divide each decibel reading by 10.

$\frac{A_2}{A_1} = 10^{(pH_1 - pH_2)}$

to get # of times more acid solution 2 is than solution 1.

Example

1a) In 1983, an earthquake measuring 5.5 on the Richter scale occurred in Columbia. In 1989, the San Francisco earthquake measured 6.9 on the Richter scale. How much more intense was the San Francisco earthquake than the Columbia earthquake?

b) Calculate the magnitude of an earthquake that is 1500 times as intense as the Columbia earthquake.

2. How much louder is a sound with an intensity of 112 dB compared to a sound with an intensity of 90 dB

b) If three different jets are flying together at an air show, each with a sound level of 120 decibels, then find the approximate total decibel level.

Example $\text{pH} = -\log\left[H^{+}\right]$ $I = I_0\left(10\right)^{P-p}$

a) A beaker of acid has a hydrogen ion concentration of 3.5×10^{-6} mol/L. Calculate the pH of the acid.

b) Solution A has a pH of 5.7. Solution B is 1260 times more **acidic** than Solution A. Find the pH of Solution B.

For you to try . . .

1. How many times more intense is an earthquake with magnitude 8.3 than one with magnitude 6.7? (Round to nearest whole number.)

2. Bob was in an earthquake of magnitude 7.1. This earthquake was 420 times more intense than a smaller earthquake that his friend Joan was in. Find the magnitude of the smaller earthquake, correct to one decimal place.

3. How many times more intense is the sound of a power saw, 120 dB, than that of a leaf rustling, 10 dB?

4. Two telephones in a home ring at the same time with a loudness of 80 decibels each. What is the decibel rating of the total loudness? (Note that 150d B is the sound of a jet engine, from 20 meters away, so the correct answer to this question is NOT 160 dB.)

5. Determine the pH of a solution, to the nearest tenth, if they hydrogen ion concentration is 3.4×10^{-4} mol/L.

6. Swimming pool water has a pH of 7.5. Sea water is about 8 times as alkaline as swimming pool water. What is the pH reading for sea water?

Answers:
1. The magnitude 8.3 earthquake is about ***40 times more intense*** than the 6.7 earthquake.
2. Magnitude of the smaller earthquake is 4.5 on Richter scale.
3. The sound of the power saw is about 10^{11} times as intense as that of a leaf rustling.
4. The total loudness is about 83 dB.
5. The solution has a pH of 3.5
6. The pH reading for sea water is about 8.4

Math History

John Napier lived from 1550-1617. He developed logarithms. In those days, logarithms were used mostly to do calculations. By using the laws of logarithms many difficult calculations could be simplified – instead of multiplying, one could use logarithms and then add, or instead of dividing, one could subtract. Here's an example:

Suppose you need to divide 217.39 by 25.461.

Logarithms helped like this:

$$\log_{10}\left(\frac{217.39}{25.461}\right) = \log_{10} 217.39 - \log_{10} 25.461$$

$$\log_{10}(quotient) = 2.337239563 - 1.405875457$$

$$\log_{10}(quotient) = 0.931364106$$

$$10^{0.931364106} = \text{quotient}$$

$$8.53815640 = \text{quotient}$$

The famous mathematician, Leonhard Euler, studied the number "*e*"

e

Leonhard Euler lived from 1707-1783. He published 530 books and papers during his lifetime. For the last 16 years of his life he was totally blind, but thanks to his phenomenal memory and ability to concentrate, he continued to generate a lot of mathematics. He would write formulas in chalk on a large slate for his secretary to copy down. He standardized these notations that you may know:

$f(x)$ for function notation

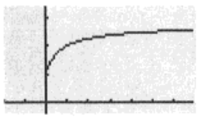
i for the imaginary unit, $\sqrt{-1}$

He came up with this formula, $e^{\pi i} + 1 = 0$, relating five of the most important numbers in mathematics.

The equation below tells us that as x gets huge, the y-values of the graph get closer and closer to the value of e:

$$e = \lim_{x \to \infty}\left(1 + \frac{1}{x}\right)^x$$

Notice that the graph is approaching a horizontal asymptote which is somewhere between 2 and 3. The table of values shows that the y values are getting closer to the actual value of e.



X	Y_1
150	2.7093
1150	2.7171
2150	2.7176
3150	2.7179
4150	2.718
5150	2.718
6150	2.7181