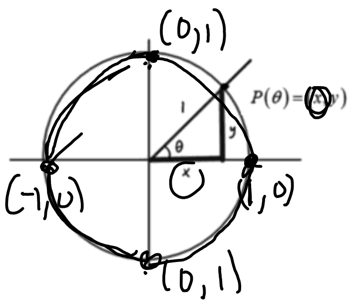
Chapter 5 Note Package
February 15, 2022 11:59 AM

Chapter 5: Trigonometric Functions and Graphs

5.1 Graphing Sine and Cosine Functions

Let's track what happens to $P(\theta)$ as θ, a standard-position angle, gets larger.

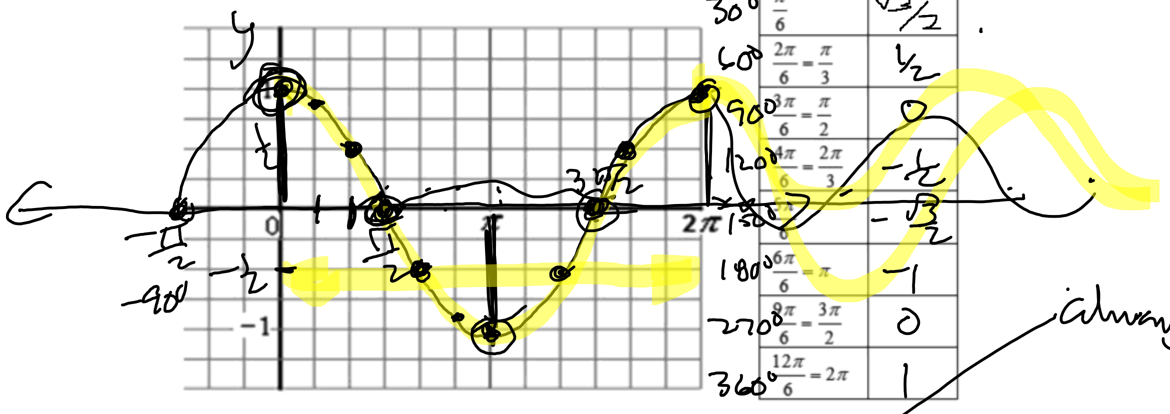


		$y = \cos\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$ 90°	cosine values (x-values) decrease from 1 to 0
Q2	As θ increases from $\frac{\pi}{2}$ to π 90° 180°	cosine values (x-values) decrease from 0 to -1
Q3	As θ increases from π to $\frac{3\pi}{2}$	cosine values (x-values) increase from -1 to 0
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	cosine values (x-values) increase from 0 to 1

$y = \cos\theta$ (also often written as $y = \cos x$)

θ	$\cos\theta$
0	1
30° $\frac{\pi}{6}$	$\sqrt{3}/2$
60° $\frac{2\pi}{6} = \frac{\pi}{3}$	1/2
90° $\frac{3\pi}{6} = \frac{\pi}{2}$	0
120° $\frac{4\pi}{6} = \frac{2\pi}{3}$	-1/2
150° $\frac{5\pi}{6}$	$-\sqrt{3}/2$
180° $\frac{6\pi}{6} = \pi$	-1
270° $\frac{9\pi}{6} = \frac{3\pi}{2}$	0
360° $\frac{12\pi}{6} = 2\pi$	1

oops



Maximum: $y=1$

Minimum: $y=-1$

Range: $\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$

Amplitude: always + → 1 unit

Domain: $\{x \mid x \in \mathbb{R}\}$

x-intercepts: $x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$ (90°, 270°)

$x = \frac{\pi}{2} + n\pi, n \in I$ (spacing between the intercepts.)

$= 90° + 180°n, n \in I$

Period:

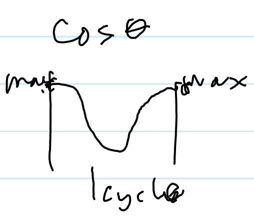
Center line equation: (mid-line) $y=0$ ← that's the equation of the x-axis.

Trig Tour

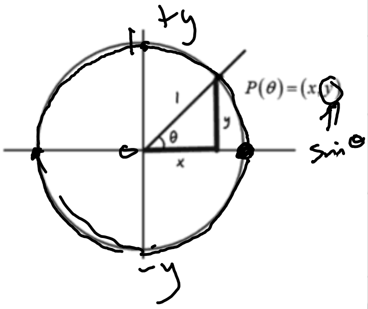
Period $= 2\pi, 360°$

The pattern that is repeated over and over is called cycle.

The period is the length of 1 cycle.

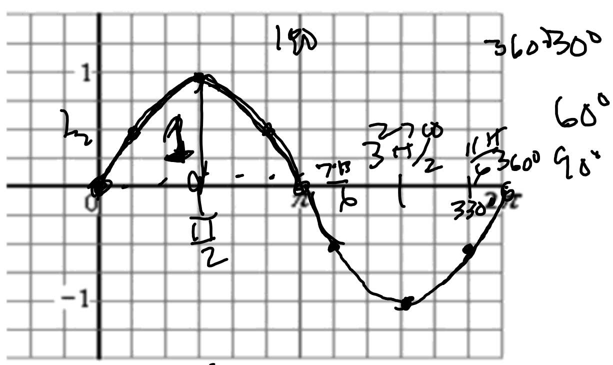


$y = \sin\theta$



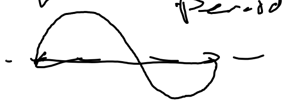
		$y=\sin\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	sine values (y-values) inc. from 0 to 1
Q2	As θ increases from $\frac{\pi}{2}$ to π	sine values (y-values) dec. from 1 to 0
Q3	As θ increases from π to $\frac{3\pi}{2}$	sine values (y-values) dec. from 0 to -1
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	sine values (y-values) inc. from -1 to 0.

$y = \sin\theta$ (also often written as $y = \sin x$)



θ	$\sin\theta$
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{2\pi}{6}=\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{3\pi}{6}=\frac{\pi}{2}$	1
$\frac{4\pi}{6}=\frac{2\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{5\pi}{6}$	$\frac{1}{2}$
$\frac{6\pi}{6}=\pi$	0
$\frac{9\pi}{6}=\frac{3\pi}{2}$	-1
$\frac{12\pi}{6}=2\pi$	0

cycle of sin θ



Period = 2π or $360°$

Maximum: $y = 1$

Minimum: $y = -1$

Range: $\{y \mid -1 \le y \le 1, y \in \mathbb{R}\}$

Amplitude: 1 unit.

Domain: $\{x \mid x \in \mathbb{R}\}$

x-intercepts: $x = 0, \pi, 2\pi, 3\pi$

$= 180°n,\ n \in I$

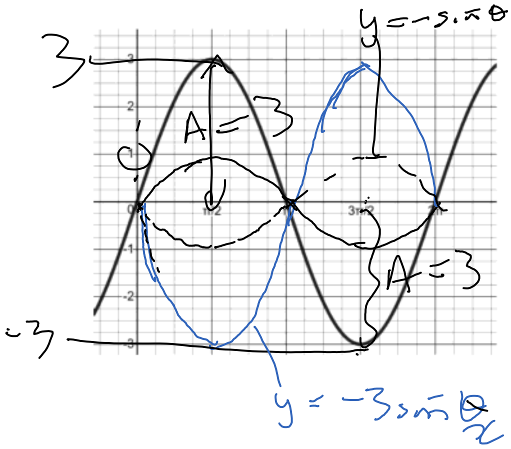
$x = n\pi,\ n \in I$

Period:

Center line equation: $y = 0$ (the x-axis)

a
A

Amplitude
is the **vertical** distance from the center line of a trigonometric graph to its maximum or minimum. The untransformed graphs of $y = \sin x$ and $y = \cos x$ have amplitude 1.



For $y = a\cos x$ or $y = a\sin x$
- vertical stretch, factor a
- amplitude = $|a|$
- if $a < 0$, graph is reflected across x-axis
- amplitude = $\frac{|\max - \min|}{2}$ $= \frac{|3 - -3|}{2} = \frac{|6|}{2} = 3.$

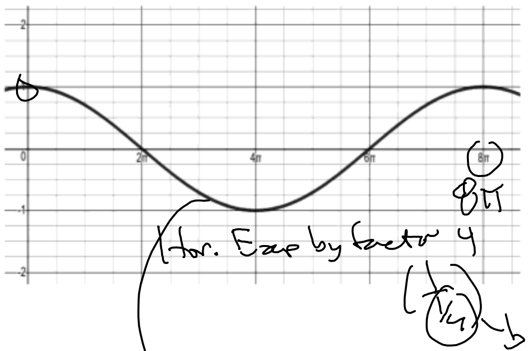
Amplitude for graph shown at left? 3 units.

Equation of the graph?

$y = 3\sin x$

Period
is the **horizontal** length of one complete cycle. The untransformed graphs of $y = \sin x$ and $y = \cos x$ have a period length of 2π (or 360°, if working in degree measure).

$y = f(x)$
$y = f(bx)$
horizontal compression of $\frac{1}{b}$ if $|b| > 1$
if $0 < |b| < 1$ a fraction
Horizontal exp by factor b.



For $y = \cos(bx)$ or $y = \sin(bx)$
- horizontal stretch, factor $\frac{1}{b}$
- period = $\frac{2\pi}{|b|}$ or $\frac{360^\circ}{|b|}$
- if $b < 0$, graph is reflected across y-axis

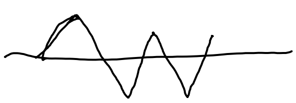
Period for graph shown at left? 8π

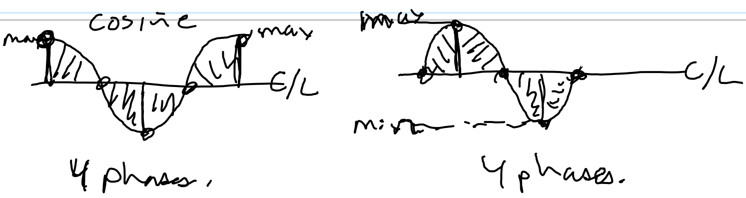
Equation of the graph?

$|b| = \frac{2\pi}{8\pi}$
$= \frac{1}{4}$

Since graph's actual period = $\frac{2\pi}{\lvert b\rvert}$, then $\lvert b\rvert = \frac{2\pi}{\text{graph's actual period}}$ If working in degrees, since graph's actual period = $\frac{360^\circ}{\lvert b\rvert}$, then $\lvert b\rvert = \frac{360^\circ}{\text{graph's actual period}}$

$y = \cos(\frac{1}{4}x)$.
Hor. exp by $\frac{1}{\frac{1}{4}}$
$= 4$.





C/L center line

When sketching a period of a trigonometric function graph, we
- multiply the period length by ¼, to determine the spacing between key points
- plot key points: maximum, minimum, and center-line
- connect key points smoothly, getting a sinusoidal shape

Period = 360°
$\frac{1}{4} \times 360^\circ = 90^\circ$

refl. over x-axis

Try
1a) $y = -3\sin(5x)$

amplitude: $a = 3$ period: $\frac{2\pi}{5}$ key point spacing: $\frac{1}{4} \times \frac{2\pi}{5} = \frac{2\pi}{20}$

1a) $y = -3\sin(5x)$

amplitude: $a = 3$ period: $\frac{2\pi}{5}$ key point spacing: $\frac{1}{4} \times \frac{2\pi}{5} = \frac{2\pi}{20} = \frac{\pi}{10}$

b) $y = -\frac{1}{4}\sin\left(\frac{1}{3}x\right)$ $b = \frac{1}{3}$

amplitude: $a = \frac{1}{4}$ period: $p = \frac{2\pi}{1/3} = 2\pi \times \frac{3}{1} = 6\pi$ key point spacing: $\frac{1}{4} \times 6\pi = \frac{6\pi}{4} = \frac{3\pi}{2}$

$b \neq$ period

Period $= \frac{2\pi}{b}$

2. Write the equation of a function with these characteristics.

a) sine function; amp = 3, period = π

a p

$y = a\sin(bx)$

$y = 3\sin(2x)$

$\pi = \frac{2\pi}{b}$

$b = 2$

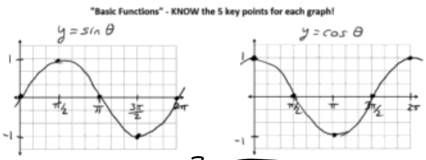
b) cosine function, amp = 2.4, period = $10\pi = \frac{2\pi}{b}$

$y = 2.4\cos\left(\frac{1}{5}x\right)$

a

$10\pi b = 2\pi$

$b = \frac{2}{10} = \frac{1}{5}$



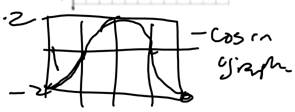
3. For each equation below, accurately sketch one ~~period~~ cycle of its graph. Give the coordinates of 5 key points.

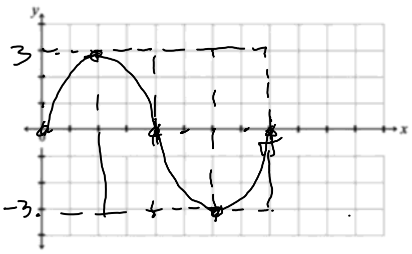
period $= \frac{2\pi}{2} = \pi$

a) $y = 3\sin(2x)$ KPS $= \frac{1}{4} \times \pi = \frac{\pi}{4}$

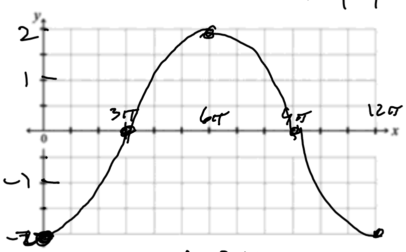
a b

b) $y = -2\cos\left(\frac{1}{6}x\right)$





x	y
0	0
$\frac{\pi}{4}$	3
$\frac{\pi}{2}$ $\frac{2\pi}{4}$	0
$\frac{3\pi}{4}$	-3
π $\frac{4\pi}{4}$	0



x	y
0	-2
3π	0
6π	2
9π	0
12π	-2

period $= \frac{2\pi}{1/6} = 2\pi \times \frac{6}{1} = 12\pi$

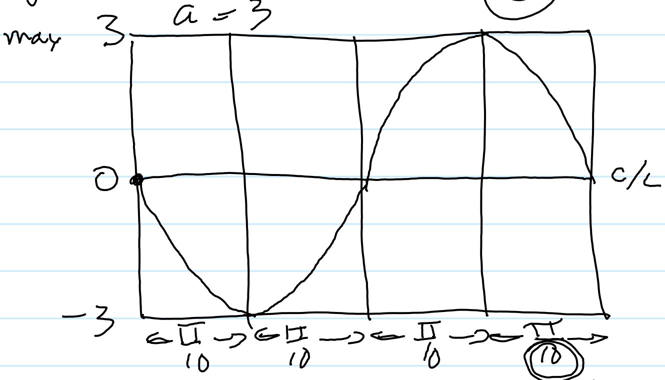
kps $= \frac{1}{4} \times 12\pi = \frac{12\pi}{4} = 3\pi$

reflected over x-axis

$\frac{4\pi}{10}$

$y = -3\sin 5x$ period $= \frac{2\pi}{5}$ kps $= \frac{\pi}{10}$

$a = 3$



period $= \frac{4\pi}{10} = \frac{2\pi}{5}$

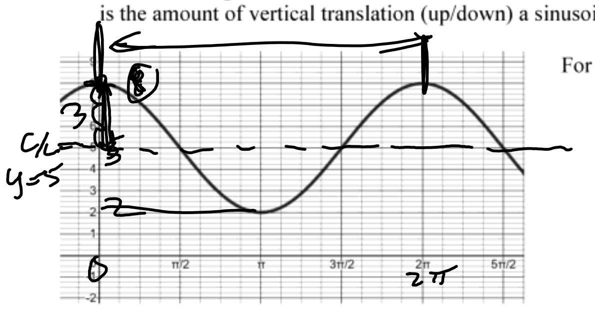
5.2 More Transformations of Sinusoidal Functions

Vertical Displacement

5.2 More Transformations of Sinusoidal Functions

Vertical Displacement

is the amount of vertical translation (up/down) a sinusoidal graph moves



For $y = a\cos bx + d$ or $y = a\sin bx + d$

- vertical displacement, d units
- center line is located at $y = d$
- when we have no equation, we can figure out the vertical displacement from the graph:

$$vertical\ disp = \frac{max + min}{2} = \frac{8+2}{2} = \frac{10}{2} = 5$$

Vertical displacement for this graph? $d = +5$

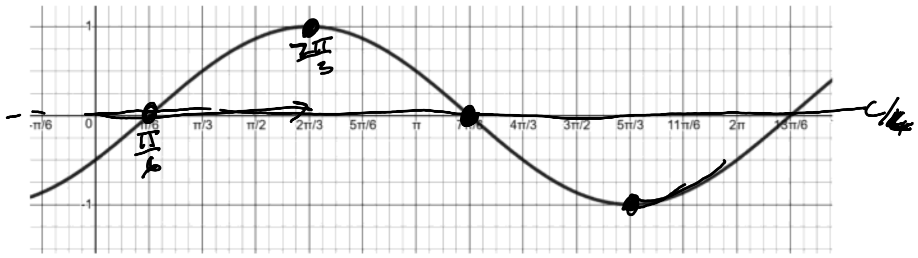
Equation of this graph? $y = a\cos bx + d$ $\quad a = 3$ $\quad$ period $= 2\pi$ $\quad b = 1$

$\boxed{y = 3\cos x + 5}$

Phase Shift

sine or cosine.

is the amount of horizontal translation (left/right) a sinusoidal graph moves



For $y = \cos(x - c)$ or $y = \sin(x - c)$

- phase shift, c units
- when we have no equation, we use the graph to find the phase shift. Choose a period of either sine or cosine that begins near the y-axis. Identify how much it has moved left/right compared to the basic (untransformed) graph.

For the graph above, find its equation in the form: $y = \sin(x - c)$ ← Starts at C/L.

For the graph above, find its equation in the form: $y = \cos(x - c)$

P.S. is $\frac{\pi}{6}$ to right. $y = \sin\left(x - \frac{\pi}{6}\right)$.

→ P.S. is $\frac{2\pi}{3}$ to right
$y = \cos\left(x - \frac{2\pi}{3}\right)$

Summary

$y = a\cos[b(x-c)]+d$

Changes period to $\frac{2\pi}{|b|}$ or $\frac{360^\circ}{|b|}$

$$y = a\sin[b(x-c)]+d$$

vertical translation.

amplitude

phase shift

$y = \sin(2x+\pi)-3$

$= \sin[2(x+\frac{\pi}{2})]-3$

Try

Determine the amplitude, period, phase shift and vertical displacement.

a) $y = 5\cos\left(3\left(x+\frac{\pi}{4}\right)\right)+2$ $b=3$ period $= \frac{2\pi}{|b|} = \frac{2\pi}{3}$

$a = 5$
period $= 2\pi/3$
p.s. $= \frac{\pi}{4}$ to the left $(c = -\frac{\pi}{4})$
vert. disp $=$ up 2 (or $+2$)

b) $y = \frac{1}{2}\cos(x+80^\circ)+5$

$a = \frac{1}{2}$
period $= 360^\circ$
p.s. $=$ 80° left $(c = -80^\circ)$
vert. disp $= +5$ or 5 up.
$(d = +5)$

c) $y = -2\sin\left(5x-\frac{\pi}{3}\right)+4$ $= -2\sin[5(x-\frac{\pi}{15})]+4$

refl. over x-axis

$b = 5$ ∴ period $= \frac{2\pi}{|b|} = \frac{2\pi}{5}$

$a = 2$
period $= 2\pi/5$
p.s. $= \frac{\pi}{15}$ to right $(c = +\frac{\pi}{15})$

vert. disp $=$ 4 units up $(d = +4)$

This is a sine function with a rational period.

Sketching a Sinusoidal Graph

Consider the equation: $y = 3\sin\left(\frac{2\pi}{12}(x+5)\right) - 1$

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

12 = period

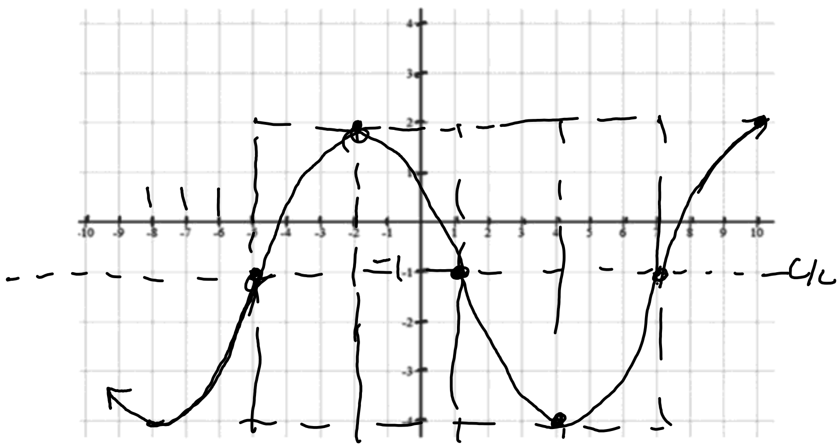
period $= \frac{2\pi}{b} = \frac{2\pi}{\pi/6} = 2\pi \times \frac{6}{\pi} = 12$ units

a) Key features:

$-5 + 12 = 7$

basic sine shape	vertical displacement	equation of center line
$y=-1$; $-1+3=2$; $-1-3=-4$; -5, -2, 1, 4, 7; 12	$d=-1$ ∴ 1 unit ↓	$y=-1$
amplitude	**maximum**	**minimum**
$a = 3$	$y = 2$	$y = -4$
period	**spacing**	**phase shift** $c = -5$
12 units.	$\frac{1}{4} \times 12 = 3$ units	∴ 5 units left

b) Accurately sketch one period of the graph. Give the coordinates of 5 key points. Include the center line on your sketch.



x	y
-5	-1
-2	2
1	-1
4	-4
7	-1
10	2

Finding the Equation of a Graph

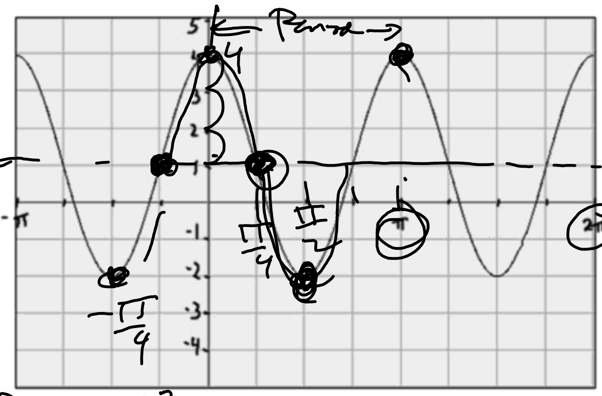
Sine and cosine graphs are both called ***sinusoidal graphs***.

- For any sinusoidal graph, it is possible to write a sine equation that creates that graph, and a cosine equation that creates that same graph.
- There are many different equations that generate the same sinusoidal graph.

Work in radians.

Example

Give two different equations that create this graph.



Maximum: $y = 4$

Minimum: $y = -2$

Center line: $y = 1$ $\left(\frac{4 + -2}{2}\right)$

Vertical displacement: $d = +1$ (1 unit up)

Amplitude: 3 units

Period: π

$|b|$ value: 2

$\pi = \frac{2\pi}{b}$ $b = 2$

$y = a\sin[b(x - c)] + d$

Possible sine equation:

P.S.: $c = -\frac{\pi}{4}$ ($\frac{\pi}{4}$ left).

$y = -4\sin[2(x - \frac{\pi}{4})] + 1$

$y = 4\sin[2(x + \frac{\pi}{4})] + 1$

← no p.s.

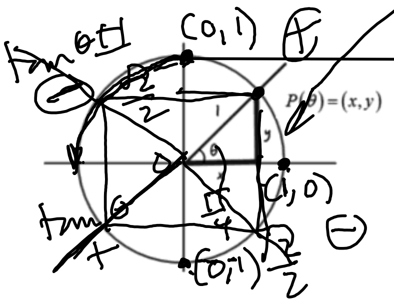
Possible cosine equation: $y = 4\cos(2x) + 1$

$y = -4\cos[2(x - \frac{\pi}{2})] + 1$

$\tan\theta = \frac{y}{x}$ on unit circle.

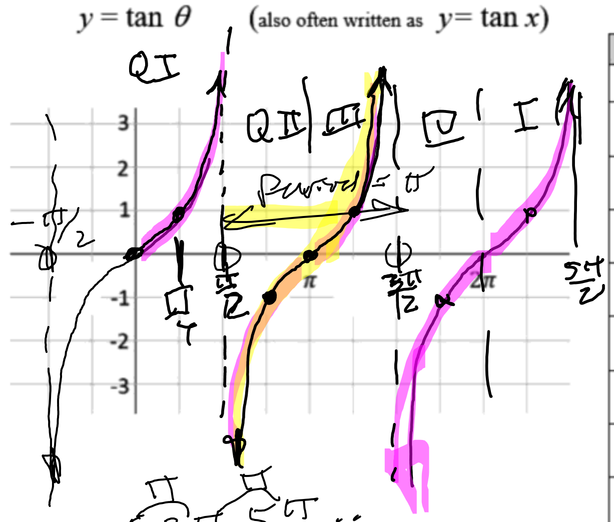
5.3 The Tangent Function

Let's track what happens to the values of $y = \tan\theta$ as θ, a standard-position angle, gets larger.



		$y = \tan\theta$
Q1	As θ increases from 0 to $\frac{\pi}{2}$	tangent values (y/x) inc. from 0 to $\frac{y}{0}$ undefined.
Q2	As θ increases from $\frac{\pi}{2}$ to π	tangent values (y/x) ↓ from undef. to 0
Q3	As θ increases from π to $\frac{3\pi}{2}$	tangent values (y/x) 0 → undef. but
Q4	As θ increases from $\frac{3\pi}{2}$ to 2π	tangent values (y/x) undef → 0

$y = \tan\theta$ (also often written as $y = \tan x$)



θ	$\tan\theta$
0	0
$\frac{\pi}{4}$	1
$\frac{2\pi}{4} = \frac{\pi}{2}$	undefined.
$\frac{3\pi}{4}$	−1
$\frac{4\pi}{4} = \pi$	0
$\frac{5\pi}{4}$	1
$\frac{6\pi}{4} = \frac{3\pi}{2}$	undef.
$\frac{7\pi}{4}$	−1
$\frac{8\pi}{4} = 2\pi$	0

$x \neq \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots$

Domain:

x-intercepts: $x = 0, \pi, 2\pi, 3\pi, -\pi$

Asymptote equations:

Period: π (180°)

Range: $\{y \mid y \in \mathbb{R}\}$

x-ints at $x = n\pi$, $n \in I$

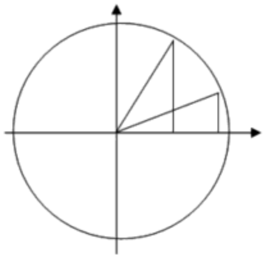
$\{x \mid x \neq \frac{\pi}{2} + n\pi, n \in I, x \in \mathbb{R}\}$

equations of asymptotes $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$ ...

in general, the equations of the asymptotes is

$x = \frac{\pi}{2} + n\pi$

The tangent graph shows how the ***slope*** of the terminal arm of a standard-position angle θ changes, as the angle increases in size.

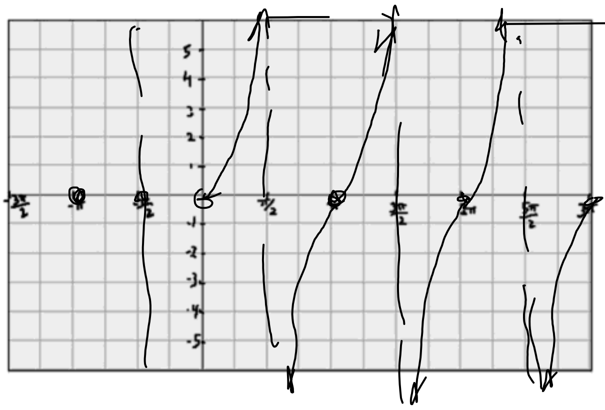


$$\tan\theta = \frac{y}{x}$$

$$slope = \frac{change\ in\ y}{change\ in\ x} = \frac{rise}{run}$$

Try

1) Sketch the graph of $y = \tan\theta$. Try to figure out points on the graph yourself (rather than just using your calculator), by using the slope idea discussed above.



Period: π

x-intercepts: $x = \frac{\pi}{2} + n\pi,\ n \in I$

Domain: $\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in R,\ n \in I$

Asymptote equations:

$x = \frac{\pi}{2} + n\pi$

2) For the graph of $y = \tan(3\theta)$

$y = f(x) \quad y = f(3x)$ $b = 3$

a) what does the "3" do?

HC by $\frac{1}{3}$

$HF = \frac{1}{3}$

HC by $\frac{1}{2}$

b) period of $y = \tan(3\theta)$

period $= \pi \times \frac{1}{3} = \frac{\pi}{3}$

c) asymptote equations for $y = \tan(3\theta)$

$y = \tan\theta$, asymptote: $x = \frac{\pi}{2} + n\pi$

$x = \left(\frac{\pi}{2}\right) + n\pi \longrightarrow x = \frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{1}{3}(n\pi)$

$= \frac{\pi}{6} + \frac{n\pi}{3} = \frac{\pi}{6} + \frac{2\pi n}{6}$

I had a 9 here

$x = \frac{1}{3}\left(\frac{\pi}{2} + n\pi\right)$

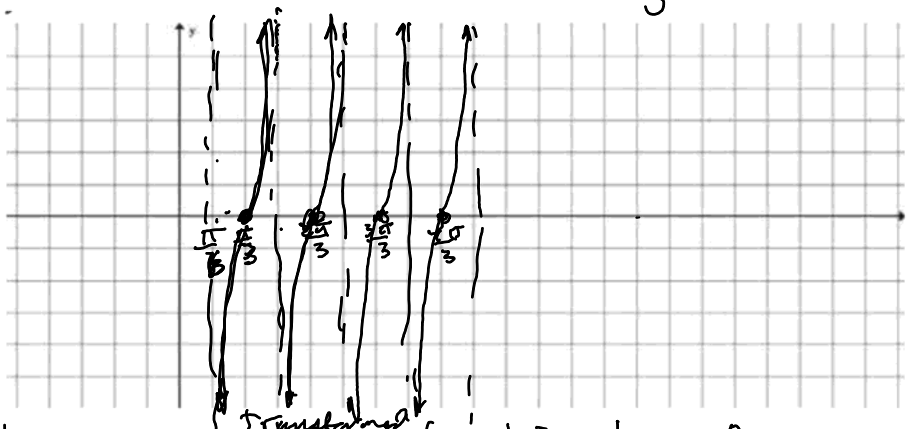
$= \frac{\pi}{6} + \frac{n\pi}{3}$

x-axis intercepts.

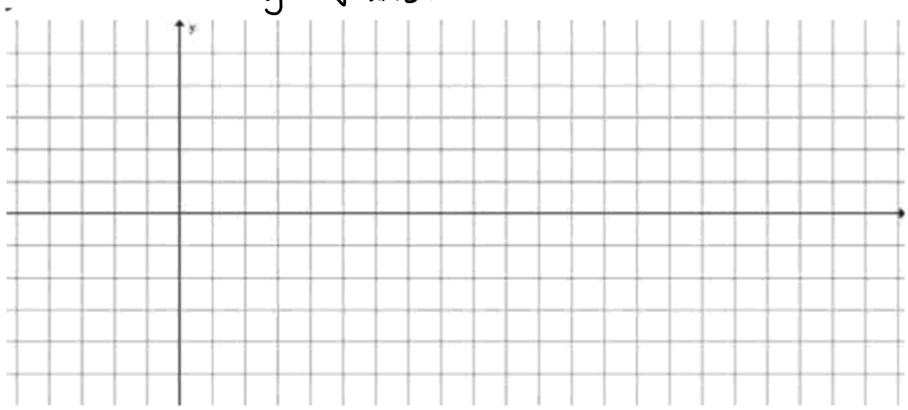
$x = (n\pi) \times \frac{1}{3} \longrightarrow x = \frac{n\pi}{3} = \frac{2\pi n}{6}$

$1 \text{ sq} = \frac{\pi}{6}$

$2 \text{ sq} = \frac{\pi}{3}$



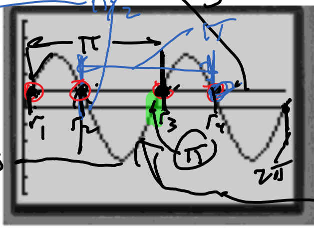
- When graphing translations of tangent functions, locate the asymptotes first.
- Do this by applying any HE or HC and any H Translations to the equations of the asymptotes of $y = \tan\theta$.

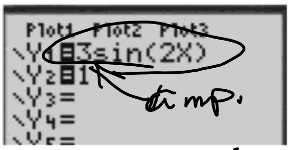


5.4 Equations and Graphs of Trigonometric Functions

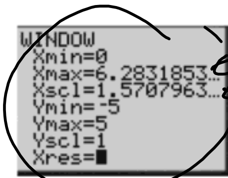
Below we see how we can solve a trigonometric equation, ***graphically***.

1.57 $\frac{\pi}{2}$ $y=1$ 3 π π r_1 r_2 r_3 r_4 π 2π 3





amp.



2π $\pi/2$

$y = 3\sin(2x)$ ← period $= \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

a) What is the equation that is being solved?

$3\sin(2x) = 1$

b) The window has been restricted to match the domain for this question. What is that domain?

$0 \le x \le 2\pi$ $b=2$ $\therefore 2\times 2$ solutions

NO $\sin^2 x$ or $\cos^2 x$

c) How many solutions are there, in this domain? Mark them on the calculator graph screenshot, shown above.

4 solutions

For a first degree sine or cos function, a equation of the form $a\sin b(x-c)+d = k$, there are $2\times b$ solution

Remember, there are two ways to solve equations GRAPHICALLY

1) Intersection Method
1) Enter the LHS of the equation as Y_1
2) Enter the RHS of the equation as Y_2
3) Set the *Xmin* and *Xmax* values using the given domain.
4) Use the "intersect" feature to find each place where the LHS = RHS. The *x*-values of the intersections are the equation's solutions.

X-intercepts (zeroes) Method
1) Collect all terms of the equation on one side of the equals sign, so it looks like
$\boxed{}$ = 0.
2) Enter the equation as Y_1
3) Set the *Xmin* and *Xmax* values using the given domain.
4) Use the "Zero" feature to find each *x*-intercept. These *x*-values are the equation's solutions.

$3\sin 2x - 1 = 1 - 1$

$3\sin 2x - 1 = 0$

graph $Y_1 = 3\sin 2x - 1$

find the zeros (x-axis intercepts!)

$0 \le x \le 2\pi$ 6.28

$3\sin 2x = 1$

$\sin 2x = \frac{1}{3}$

$2x = \sin^{-1}\left(\frac{1}{3}\right)$

$2x = 0.339836\ldots$

$x = \frac{0.339836\ldots}{2}$

$x = 0.169918$

$x = 0.1699$ 0.1699 0.1699 = ref∠

the period $= \pi$

$x = 0.1699 + \pi$

$= 3.3115\ldots \to \boxed{3.3115}$

QI

In QII, $2x = \pi - 0.339836$

$2x = 2.80175\ldots$

$x = \frac{2.80175\ldots}{2}$

With a graphing general

With a graphing calculator, you can do the following:

(General solution $x = 0.1699 + n\pi$, $n \in I$)

$2x = 2.80175..$
$x = \frac{2.80175..}{2}$
$= 1.40087$
$\boxed{x = 1.4009}$ ✓

and $x = 1.4008 + \pi$
$\boxed{x = 4.5425}$ ✓

(General solution $x = 1.4009 + n\pi$, $n \in I$)

- graph $Y_1 = 3\sin(2x)$
$Y_2 = 1$

- find the x-values at the points of intersection of the two graphs.

$\boxed{2nd}$ $\boxed{\text{trace}}$ (calc) Choose 5: intersect

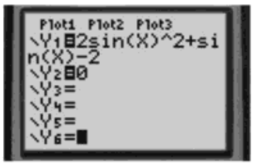
- Move cursor close to the intersection point
- press $\boxed{\text{enter}}$ 3 times

Using the x-intercepts method, you only graph 1 equation and then you find the x-intercepts using $\boxed{\text{Calc}}$ 2: zero (zeros)

To use the CALC 2: zero feature, you need to set a left bound and a right bound, then "guess" by hitting $\boxed{\text{enter}}$

Try
Solve ***graphically***, correct to 1 decimal place. Include a sketch of the graph with the solutions marked on it.

$$2\sin^2 x + \sin x - 2 = 0, \text{ for } 0^\circ \le x \le 720^\circ$$

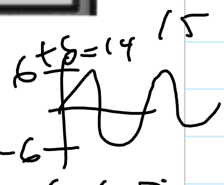


~~*Example*~~ you do this one.

Consider the trigonometric equation $6\sin\left(\frac{\pi}{4}x\right)+8=10$

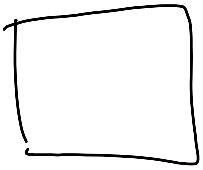
$y_1=$ type $6\sin\frac{\pi}{4}x$ -8 $y_2=10$

$a=6$ $6+8=14$ 15 $-6+8=2$ -2



a) Solve ***graphically*** for $0 \le x < 2\pi$, correct to 4 decimal places. Include a sketch of the graph with the solutions marked on it. rad - ms

$6\sin\left(\frac{\pi}{4}x\right) = 2$



b) Find the general solution, ***algebraically***, correct to 4 decimal places.

$x = 0.4322$, $x = 3.5673$

$6\sin\left(\frac{\pi}{4}x\right)+8=10$
$-8 \qquad -8$

$\frac{6\sin\left(\frac{\pi}{4}x\right)}{6} = \frac{2}{6}$

$\sin\left(\frac{\pi}{4}x\right) = \frac{1}{3}$

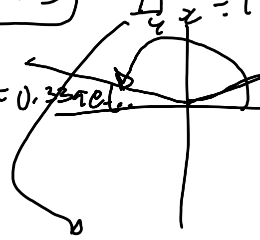
$\frac{\pi}{4}x = \sin^{-1}\left(\frac{1}{3}\right) = 0.3398369$

$4 \times \frac{\pi}{4}x = 0.3398369 \times 4$

$\frac{\pi x}{\pi} = \frac{0.3398369 \times 4}{\pi} = 0.43269$

$\frac{\pi}{4}x = \pi - 0.3398...$

$\theta_R = 0.3398...$

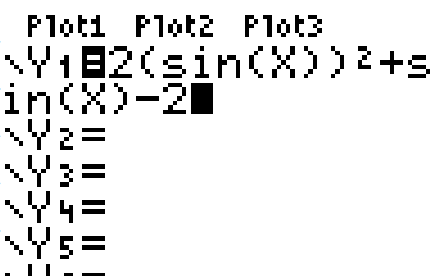


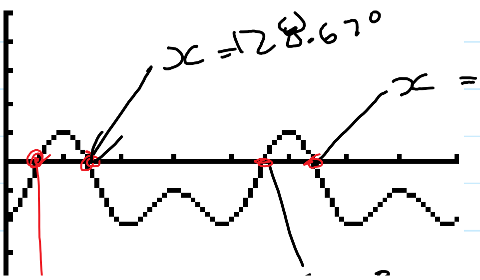
$0.3398369 = \theta_R$

$\frac{\pi}{4}x = 2.8017557$

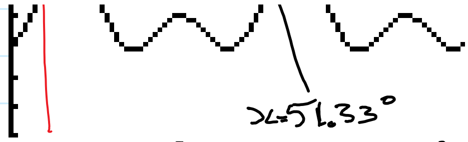
$x = \frac{2.8017557 \times 4}{\pi}$

$x = 3.5673$





\Y4=
\Y5=
\Y6=



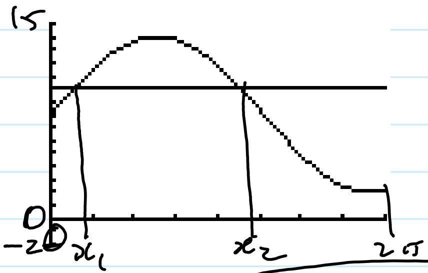
$x = 51.33°$

$x = 51.33° + 360° = 411.33°$

Or $2\sin^2 x + \sin x - 2 = 0$
$+2 \quad +2$

$\underbrace{2\sin^2 x + \sin x}_{Y_1 =} = \underbrace{2}_{Y_2 = 2}$

Graph $Y_1 = 2\sin^2 x + \sin x$
$Y_2 = 2$
find the intersection points



WINDOW
Xmin=0
Xmax=6.2831853… 2π
Xscl=.78539816…
Ymin=-2
Ymax=15
Yscl=1
Xres=■

who

$b = \frac{\pi}{4} \qquad p = \frac{2\pi}{\frac{\pi}{4}} = 2\pi \times \frac{4}{\pi} = 8$

Plot1 Plot2 Plot3
\Y1■6sin(πX/4)+8
■
\Y2■10
\Y3=
\Y4=
\Y5=
\Y6=

Q: What's the period of that?

$Y_1 = a\sin(bx) + d = 6\sin\left(\frac{\pi}{4}x\right) + 8$

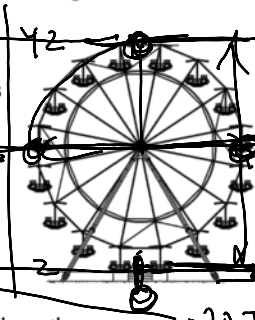
$Y_2 = 10$

$x_1 = 0.43269379 \qquad x_2 = 3.5673062$
$x_1 = 0.43 \qquad = 3.57$

metres

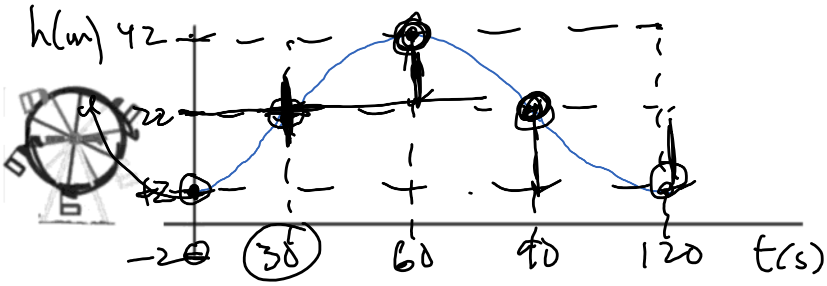
Example

Suppose the pictured Ferris wheel has diameter 40 ~~meters~~ metres, and the height of the ~~seat~~ where you first get on is 2 ~~meters~~ above the ground. This wheel takes 2 minutes to rotate and travels at a constant speed.



- Minimum height? 2 m
- Maximum height? 42 m
- Center line height? 22m
- Period length in seconds? $p = 2\text{ min} = 120\text{ (s)}$

a) Sketch a complete period of the graph, showing the height of a passenger above the ground as a function of time, in seconds. Give the coordinates of 5 key points.



t (x)	h (m) (y)
0	2
30	22
60	42
90	22
120	2

1 rotation takes 120s

1/4 rotation takes 30s ($\frac{1}{4} \times 30$)

$p = \frac{2\pi}{b} = 120$

b) Create a sinusoidal equation for this graph. → sine or cosine, for − $b = \boxed{\frac{2\pi}{120}}$

$|a| = 20$ m

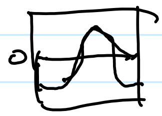
$d = +22$

$h(t) = -20\cos\left(\frac{2\pi}{120}x\right) + 22$

or $y = -20\cos\left(\frac{2\pi}{120}x\right) + 22$

$y = -(a)\cos bx + d$ 20 t $+22$ $h(t)$

$y = -\cos x$



c) How high above the ground is a passenger 12 seconds after getting on, correct to one decimal place?

$h(12) = -20\cos\left(\frac{2\pi}{120}(12)\right) + 22$

Calc: value $= 5.82$ m

X=12 Y=5.8196601

d) During the first rotation of the Ferris wheel, what is the first time that the passenger reaches a height of 30 ~~meters~~ metres above the ground? Solve this ***graphically***.

$30 = -20\cos\left(\frac{2\pi}{120}t\right) + 22$ x

$Y_1 =$

Graph $Y_2 = 30$,

find the intersection

x = 37.859372

Y = 30

$\therefore t = 37.86$ s

As a positive sine function.

$h(t) = 20\sin\left(\frac{2\pi}{120}(x - 30)\right) + 22$