C_21 Key and Chapter 8 Review

November-19-13 3:24 PM

Chapter 8 Review Worksheet

Name: _____

1. Write in logarithmic form: a) $5^x = 27$ b) $x^w = t$

2. Write in exponential form: a) $\log_6 r = t$ b) $\log_b c = a$

3. Use logarithm laws to re-write each expression so it uses only one "log".
   a) $\log 5 + \log r$

   b) $\log x - \log 11$

   c) $\dfrac{\log t}{\log r}$

4. Evaluate without using your calculator:
   a) $\log_5 5^3$ b) $\log_8 8^4$

   c) $\log_x x^2$ d) $\log 10^{8.2}$

   e) $10^{\log 4}$ f) $\log_2 1$

5. Write an equation and solve it to find the amount of money you have after 5 years, if you invest $2000 and receive 7% annual interest, compounded quarterly.

6. You find 30 fruit flies in your kitchen. Suppose their population triples every 4 days. Write and solve an equation to find how many days it takes until there are 1500 fruit flies.

7. Suppose that a laboratory has 50 g of a radioactive element that has a half-life of 2 weeks. How long will it take until this sample is reduced to 4 g?

8. You've been in an earthquake measuring 6.4 on the Richter scale and want to figure out how much more intense that is than a 3.9 one a friend was in. Write and solve the equation to do this.

9. State the domain and the x-intercept for the graph of: $y = \log_4(2x-5)-2$

10. For every 100 meters that a balloon rises, the atmospheric pressure is reduced by 1%. At what balloon height is the atmospheric pressure 15% of the pressure at the earth's surface?

11. Solve for x.

a) $\log_3 x = -4$

b) $\log x - \log 7 = \log 18$

c) $\log_4 x + \log_4(x+1) = 2$

d) $3^{x-2} = 5^{2x+1}$

12. Evaluate: $\log_3 59.2$

13. Determine the domain of $y = \log(-x)$.

14. The point (2, 9) is on the graph of $y = b^x$. What point must be on the graph of $y = \log_b x$?

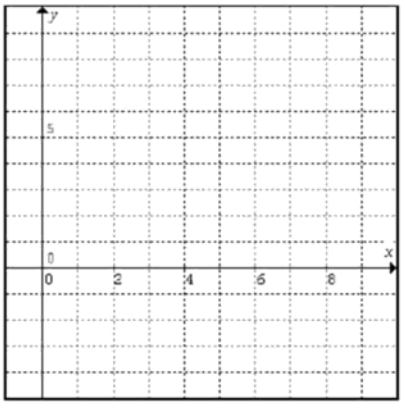
15. Solve: $\log_2(3-2x) - \log_2(2-x) = \log_2 3$

16. If $\log MN = 8$ and $\log M = -4$, determine N.

17a) Fill in the table below and use the values to sketch the graph of $y = \log_3 x$ onto the grid below. Sketch in its asymptote.

$y = \log_3 x$

x	y



domain

asymptote equation

b) Suppose that $y = \log_3 x$ is changed to $y = 4\log_3(x-1)$. What two transformations occur?

x	y

c) Transform the points you found above for $y = \log_3 x$ and fill in this new table for the equation $y = 4\log_3(x-1)$.

d) Sketch the graph of $y = 4\log_3(x-1)$ onto the grid above. Include its asymptote.

e) For $y = 4\log_3(x-1)$ what are its:

domain

asymptote equation

Answers

1a) $\log_5 27 = x$ b) $\log_x t = w$

2a) $6^t = r$ b) $b^a = c$

3a) $\log 5r$ b) $\log\left(\frac{x}{11}\right)$

c) $\log_r t$

4a) 3 b) 4 c) 2
d) 8.2 e) 4 f) 0

5. $A = 2000\left(1+\frac{0.07}{4}\right)^{20} = \$2829.56$

6. $1500 = 30(3)^{t/4}$

$50 = (3)^{t/4}$ *(divide both sides by 30, to isolate exponential term)*

$\log 50 = \frac{t}{4}\log 3$ *(log both sides; use power law)*

$4\log 50 = t\log 3$ *(eliminate fraction)*

$t = \frac{4\log 50}{\log 3} \doteq 14.24$ days

(divide and evaluate on calculator)

7. $4 = 50(0.5)^{t/2}$, $t = 7.3$ weeks
(Solving process is very similar to #6.)

8. $I = I_0(10)^{(6.4-3.9)}$, so $I \doteq 316 I_0$.
The stronger earthquake is about 316 times more powerful than the weaker one.

9. Domain: $x > \frac{5}{2}$
Recall, argument must be greater than zero:

$2x - 5 > 0, \quad x > \frac{5}{2}$

x-intercept: (10.5, 0)

$0 = \log_4(2x-5) - 2$ *(let $y = 0$)*

$2 = \log_4(2x-5)$ *(isolate log term)*

$4^2 = 2x - 5$ *(change to exponential form)*

$x = 10.5$ *(solve)*

10. $15 = 100(0.99)^{h/100}$. Solve, h = 19 000 m

11a) $x = 3^{-4} = \frac{1}{81}$ b) $x = 126$

c) $\log_4(x^2 + x) = 2$,

$4^2 = x^2 + x$

$x^2 + x - 16 = 0$

Since this doesn't factor, solve using the quadratic formula. We get x = 3.53 and x = –4.53. We reject –4.53, x = 3.53 is the only solution.

d) $(x-2)\log 3 = (2x+1)\log 5$

(log both sides, use power rule)

$x\log 3 - 2\log 3 = 2x\log 5 + 1\log 5$

(distribute)

$x\log 3 - 2x\log 5 = \log 5 + 2\log 3$

(collect x terms on one side)

$x(\log 3 - 2\log 5) = \log 5 + 2\log 3$

(factor out x as GCF)

$x = \frac{\log 5 + 2\log 3}{\log 3 - 2\log 5}$

(divide)

$x = -1.795372332 \approx -1.80$

(evaluate on calculator)

12. Use change of base law:

$\log_3 59.2 = \frac{\log 59.2}{\log 3} = 3.71$

13. We know the argument must be greater than 0: $-x > 0$
Divide both sides by -1, to get $x < 0$.

14. (9, 2)
(Because these are inverses of one another.)

15. no solution (reject $x = 3$)

16. $\log MN = 8$ $\log N = 12$

$\log M + \log N = 8$ $N = 10^{12}$

$-4 + \log N = 8$

Chapter 8 Review Worksheet

Name: Solutions

1. Write in logarithmic form: a) $5^x = 27$ b) $x^w = t$

$\log_5 27 = x$ $\log_x t = w$

2. Write in exponential form: a) $\log_6 r = t$ b) $\log_b c = a$

$6^t = r$ $b^a = c$

3. Use logarithm laws to re-write each expression so it uses only one "log".

a) $\log 5 + \log r$ $= \log(5r)$ (Product Law)

b) $\log x - \log 11$ $= \log\left(\frac{x}{11}\right)$ (Quotient Law)

c) $\frac{\log t}{\log r}$ $= \log_r t$ (Change of Base Law)

4. Evaluate without using your calculator:

a) $\log_5 5^3$ $= 3\log_5 5$ (Power Law) $= 3(1) = 3$

b) $\log_8 8^4$ $= 4\log_8 8$ (Power Law) $= 4(1) = 4$

c) $\log_x x^2$ $= 2\log_x x$ (Power Law) $= 2(1) = 2$

d) $\log 10^{8.2}$ $= 8.2 \log 10$ (Power Law) $= 8.2(1) = 8.2$

e) $10^{\log 4} = 4$

$\log_{10} 4$ = the exponent needed to change the base 10 into the argument 4

10 is being raised to the exponent that is able to change 10 into 4

f) $\log_2 1 = 0$ (because $2^0 = 1$)

10 is being raised to the exponent that is able to change 10 into 4

$\Rightarrow 10^{\square} = 4$

5. Write an equation and solve it to find the amount of money you have after 5 years, if you invest $2000 and receive 7% annual interest, compounded quarterly.

↳ 4 times a year

$A = 2000(1 + 0.0175)^{20}$

$\doteq \$2829.56$

$P = 2000$

$i = \frac{0.07}{4} = 0.0175$

$n = 4(5) = 20$ periods

6. You find 30 fruit flies in your kitchen. Suppose their population triples every 4 days. Write and solve an equation to find how many days it takes until there are 1500 fruit flies.

$A = 30(3)^{t/4}$

$\frac{1500}{30} = \frac{30}{30}(3)^{t/4}$

$50 = 3^{t/4}$

$\log 50 = \log 3^{t/4}$

$4 \times \log 50 = \left(\frac{t}{4}\log 3\right) \times 4$

$4\log 50 = t\log 3$

$t = \frac{4\log 50}{\log 3} \doteq$ 14.24 days

$A_0 = 30$

$b = 3$

$p = 4$ days

7. Suppose that a laboratory has 50 g of a radioactive element that has a half-life of 2 weeks. How long will it take until this sample is reduced to 4 g?

$\frac{4}{50} = \frac{50}{50}\left(\frac{1}{2}\right)^{t/2}$

$\frac{4}{50} = \left(\frac{1}{2}\right)^{t/2}$

$\log 0.08 = \log(0.5)^{t/2}$

$2 \times (\log 0.08) = \left(\frac{t}{2}\log 0.5\right) \times 2$

$2\log 0.08 = t\log 0.5$

$t = \frac{2\log 0.08}{\log 0.5}$

$t \doteq 7.3$ weeks

$A_0 = 50$

$b = \frac{1}{2}$

$p = 2$ weeks

8. You've been in an earthquake measuring 6.4 on the Richter scale and want to figure out how much more intense that is than a 3.9 one a friend was in. Write and solve the equation to do this.

$I = I_0 10^{R-r}$

$I = I_0 (10)^{6.4-3.9}$

$I = I_0 (10)^{2.5}$

$I \doteq I_0 (316.23)$

Yours is about 316 times more intense than the earthquake the friend was in.

9. State the domain and the x-intercept for the graph of: $y = \log_4(2x-5) - 2$

argument > 0

$2x - 5 > 0$

$2x > 5$

$x > \frac{5}{2}$

$D = \{x \mid x > \frac{5}{2}, x \in \mathbb{R}\}$

x-intercept, let $y = 0$

$0 = \log_4(2x-5) - 2$

$2 = \log_4(2x-5)$

$4^2 = 2x - 5$

$16 = 2x - 5$

$2x = 21$

$x = \frac{21}{2}$

$(10.5, 0)$

10. For every 100 meters that a balloon rises, the atmospheric pressure is reduced by 1%. At what balloon height is the atmospheric pressure 15% of the pressure at the earth's surface?

$A = A_0(b)^{t/p}$

$\frac{15}{100} = \frac{100}{100}(0.99)^{t/100}$

$0.15 = (0.99)^{t/100}$

$\log 0.15 = \log(0.99)^{t/100}$

$100 \times \left(\frac{t}{100}\log 0.99\right) = (\log 0.15) \times 100$

$\frac{t \log 0.99}{\log 0.99} = \frac{100 \log 0.15}{\log 0.99}$

$t \doteq 18,876$ m

$\Rightarrow$ 19,000 m, if we round to nearest thousand

11. Solve for x.

a) $\log_3 x = -4$

$3^{-4} = x$, $x = \frac{1}{3^4} = \frac{1}{81}$ ← Hey, that's my favorite fraction!

b) $\log x - \log 7 = \log 18$

$\log\left(\frac{x}{7}\right) = \log 18$ (Quotient Law)

$\Rightarrow \frac{x}{7} = 18$

$\frac{7}{1} \times \left(\frac{x}{7}\right) = (18) \times 7$

$x = 126$

c) $\log_4 x + \log_4(x+1) = 2$

$\log_4 x(x+1) = 2$ (Product Law)

$4^2 = x(x+1)$

$16 = x^2 + x$

$x^2 + x - 16 = 0$

$x = \frac{-1 \pm \sqrt{1^2 - (4)(1)(-16)}}{2(1)}$

$x = \frac{-1 \pm \sqrt{65}}{2}$

$x \doteq 3.53$, ~~-4.53~~ reject, makes argument negative

d) $3^{x-2} = 5^{2x+1}$

$\log 3^{x-2} = \log 5^{2x+1}$

$(x-2)\log 3 = (2x+1)\log 5$ (Power Law)

$x\log 3 - 2\log 3 = 2x\log 5 + 1\log 5$ (distribute)

$x\log 3 - 2x\log 5 = \log 5 + 2\log 3$

$x(\log 3 - 2\log 5) = \log 5 + 2\log 3$

$x = \frac{\log 5 + 2\log 3}{\log 3 - 2\log 5}$

$x = -1.79537\ldots$

$x \doteq -1.80$

12. Evaluate: $\log_3 59.2$ $= \dfrac{\log 59.2}{\log 3}$

$\doteq \boxed{3.71}$

13. Determine the domain of $y = \log(-x)$.

Argument > 0

$-x > 0$

$-1 \quad -1$

$x < 0$

$\boxed{D = \{x \mid x < 0, x \in \mathbb{R}\}}$

14. The point (2, 9) is on the graph of $y = b^x$. What point must be on the graph of $y = \log_b x$?

Since these are inverses, $\boxed{(9, 2)}$ is on graph.

15. Solve: $\log_2(3-2x) - \log_2(2-x) = \log_2 3$

$\log_2\left(\dfrac{3-2x}{2-x}\right) = \log_2 3$ (Quotient Law)

$\Rightarrow \dfrac{(2-x)}{1}\left[\dfrac{3-2x}{2-x}\right] = [3](2-x)$

$3-2x = 3(2-x)$

$3-2x = 6-3x$

$3-2x+3x = 6$

$x = 6-3$

$x = 3$

But, if $x = 3$ is substituted in, it makes negative arguments.

$\Rightarrow$ no solution

16. If $\log MN = 8$ and $\log M = -4$, determine N.

$\log MN = \log M + \log N$ (Product Law)

$= -4 + \log N$ (substitution)

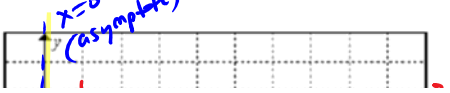
$8 = -4 + \log N$

$12 = \log N$

$10^{12} = N$

$\boxed{N = 10^{12}}$

17a) Fill in the table below and use the values to sketch the graph of $y = \log_3 x$ onto the grid below. Sketch in its asymptote.



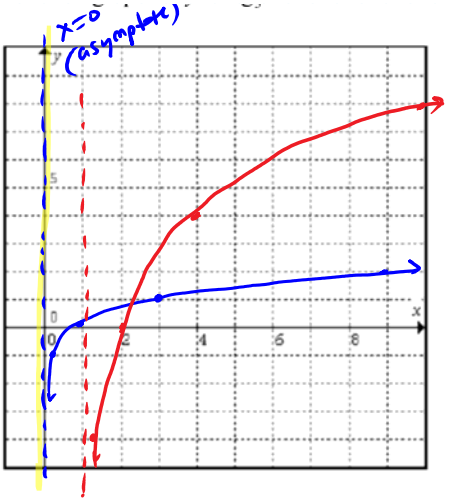
grid below. Sketch in its asymptote.

$y = \log_3 x$

choose powers of 3

x	y
$\frac{1}{3}$	-1
1	0
3	1
9	2

$\log_3(\frac{1}{3}) = -1$
$\log_3 1 = 0$
$\log_3 3 = 1$
$\log_3 9 = 2$



domain $\{x \mid x > 0,\ x \in \mathbb{R}\}$

asymptote equation $x = 0$

b) Suppose that $y = \log_3 x$ is changed to $y = 4\log_3(x-1)$. What two transformations occur?

VS 4, right 1

Right 1 VS 4

x	y
$1\frac{1}{3}$	-4
2	0
4	4
10	8

c) Transform the points you found above for $y = \log_3 x$ and fill in this new table for the equation $y = 4\log_3(x-1)$.

d) Sketch the graph of $y = 4\log_3(x-1)$ onto the grid above. Include its asymptote.

e) For $y = 4\log_3(x-1)$ what are its:

domain $\{x \mid x > 1,\ x \in \mathbb{R}\}$

asymptote equation $x = 1$