Chapter 6 Hand-in Assignment – Trigonometric Identities

Name: Key

You are expected to show your work!!

1. Simplify each expression.

a) $\dfrac{\tan x}{\sec x}$ $= \sin x$

b) $\dfrac{1}{\tan x \csc x}$ $= \cos x$

c) $\dfrac{1-\cot x}{\tan x - 1}$ $= \cot x$

d) $\dfrac{1+\cot^2 x}{\cot^2 x}$ $= \sec^2 x$

e) $\sec x \cos x + \dfrac{\cos^2 x}{\sin^2 x}$ $= \csc^2 x$

f) $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x}$ $= 1$

2. Verify the following identity algebraically, for $x = \frac{\pi}{4}$

$$\frac{1-\sin x}{\cos x} = \frac{\cos x}{1+\sin x}$$

$$\frac{1-\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{\pi}{4}\right)} \overset{?}{=} \frac{\cos\left(\frac{\pi}{4}\right)}{1+\sin\left(\frac{\pi}{4}\right)}$$

$$\frac{1-\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \overset{?}{=} \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$$

$$0.41423\ldots = 0.41423\ldots \checkmark$$

3. Write each expression as a single trigonometric function. SHOW WORK!

a) $\cos 20^\circ \cos 5^\circ + \sin 20^\circ \sin 5^\circ$ $= \cos 15^\circ$

b) $2\cos^2\left(\frac{\pi}{5}\right) - 1$ $= \cos \frac{2\pi}{5}$

c) $2\sin 7x \cos 7x$ $= \sin 14x$

SHOW WORK

4. Use identities and special angle values to determine the exact value of each trigonometric expression.

a) $\cos 195^\circ$ $= \dfrac{-\sqrt{2}-\sqrt{6}}{4}$

b) $\sin 255^\circ$ $= \dfrac{-\sqrt{2}-\sqrt{6}}{4}$

SHOW

5. If ∠A is in quadrant I, ∠B is in quadrant III, and $\sin A = \frac{7}{25}$ and $\cos B = -\frac{8}{17}$, use identities to evaluate each of the following: SHOW WORK.

a) $\sin(A-B)$

$= \dfrac{304}{425}$

b) $\cos 2B$ $= \dfrac{-161}{289}$

c) $\sin 2A$ $= \dfrac{336}{625}$

methods may vary

6. Prove the following identities:

a) $$\frac{1+\cos 2x}{\sin 2x} = \cot x$$

$$\frac{1+(2\cos^2 x-1)}{2\sin x\cos x}$$

$$\frac{2\cos^2 x}{2\sin x\cos x}$$

$$\frac{2\cos x}{\sin x}$$

$$\cot x = \cot x \quad ✓$$

b) $$\frac{1-\sin^2 x-2\cos x}{\cos^2 x-\cos x-2} = \frac{1}{1+\sec x}$$

$$\frac{\cos^2 x-2\cos x}{(\cos x-2)(\cos x+1)}$$

$$\frac{\cos x(\cancel{\cos x-2})}{(\cancel{\cos x-2})(\cos x+1)}$$

$$\frac{\cos x}{\cos x+1}$$

$$\frac{\cos x\times\frac{1}{\cos x}}{(\cos x+1)\times\frac{1}{\cos x}}$$

$$\frac{1}{\frac{\cos x}{\cos x}+\frac{1}{\cos x}}$$

$$\frac{1}{1+\sec x} = \frac{1}{1+\sec x} \quad ✓$$

c) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

$$\frac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\sin x}{1}}$$

$$\frac{\frac{1+\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \frac{\sin x \cos x}{\cos x}}$$

$$\frac{\frac{1+\cos x}{\sin x}}{\frac{\sin x(1+\cos x)}{\cos x}}$$

$$\frac{\cancel{1+\cos x}}{\sin x} \times \frac{\cos x}{\sin x(\cancel{1+\cos x})}$$

$$\frac{\cos x}{\sin x} \times \frac{1}{\sin x}$$

$$\frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x}{\sin^2 x} \quad \Bigg| \quad \frac{\cos x}{\sin^2 x}$$

d) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x} \quad \Bigg| \quad \frac{\cos x(1+\sin x)}{(1-\sin x)(1+\sin x)}$$

$$\frac{1+\sin x}{\cos x} \quad \Bigg| \quad \frac{\cos x(1+\sin x)}{1-\sin^2 x}$$

$$\frac{\cos x(1+\sin x)}{\cos^2 x}$$

$$\frac{1+\sin x}{\cos x} = \frac{1+\sin x}{\cos x} \checkmark$$

SHOW WORK.

7. Solve each equation algebraically, over the given domain. You will need to use identities!

a) $\sin 2x + \cos x = 0$, for $0^\circ \leq x < 360^\circ$.

$x = 90^\circ, 270^\circ, 210^\circ, 330^\circ$

b) $\sin^2 x = \cos x - \cos 2x$, for $0 \leq x < 2\pi$

$x = 0, \frac{\pi}{2}, \frac{3\pi}{2}$