

## Chapter 5 Hand-in Assignment – Trigonometric Graphs and Applications

Name: \_\_\_\_\_

*Don's Key*

1. Fill in the table below, stating each function's amplitude, period, phase shift and vertical displacement.

Equation	Amplitude	Period	Phase Shift (how much, and in which direction)	Vertical displacement (how much, and in which direction)	Maximum	Minimum
$y = 4 \cos\left(x + \frac{\pi}{4}\right) - 3$	4	$2\pi$	$\frac{\pi}{4}$ left	3 units down	1	-7
$y = -7 \sin(2x - \pi) + 5$ <i><math>y = -7 \sin(2(x - \frac{\pi}{2})) + 5</math></i>	7	$\pi$	$\frac{\pi}{2}$ right	5 units up	12	-2
$y = 2 \cos\left(\frac{1}{2}(x + 30^\circ)\right) - 1$	2	$720^\circ$	$30^\circ$ left	1 unit down	1	-3

2. Write an equation for a function with the given characteristics:

a) A **sine** function with amplitude 5, period  $\pi$ , phase shift  $\frac{\pi}{3}$  left, and vertical displacement 2 down

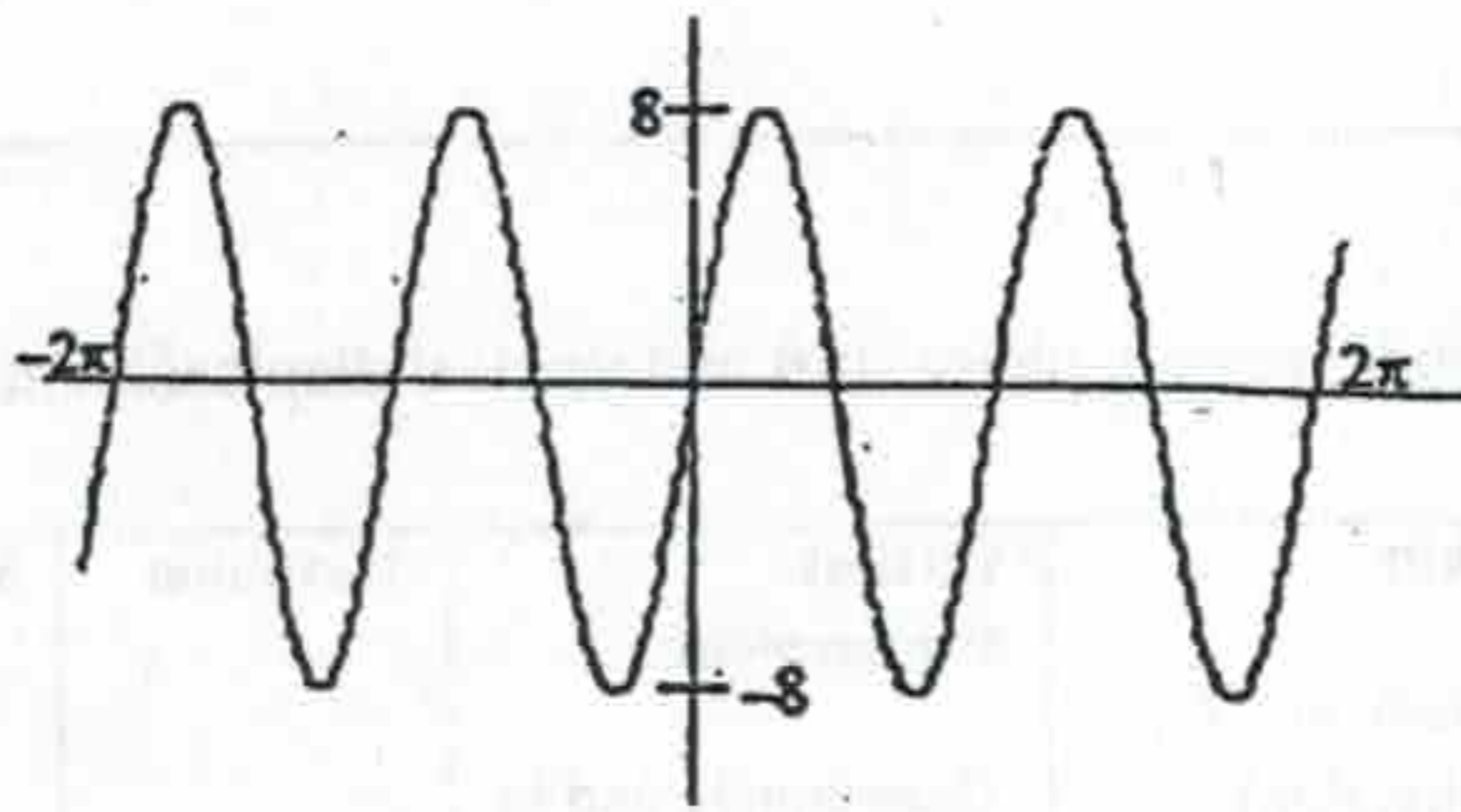
$$y = -5 \sin\left(\frac{2}{3}\left(x + \frac{\pi}{3}\right)\right) - 2$$

b) A **cosine** function with amplitude 2, period  $3\pi$ , phase shift 5 right, and vertical displacement 4 up

$$y = 2 \cos\left(\frac{2}{3}(x - 5)\right) + 4$$



3. For the graph shown below, find:



Amplitude

8

Period

$\pi$

A correct equation for it.

$$y = -8 \sin(2x)$$

4a) State the **range** of the graph for the function  $y = 8 \cos(x - 45^\circ) + 5$

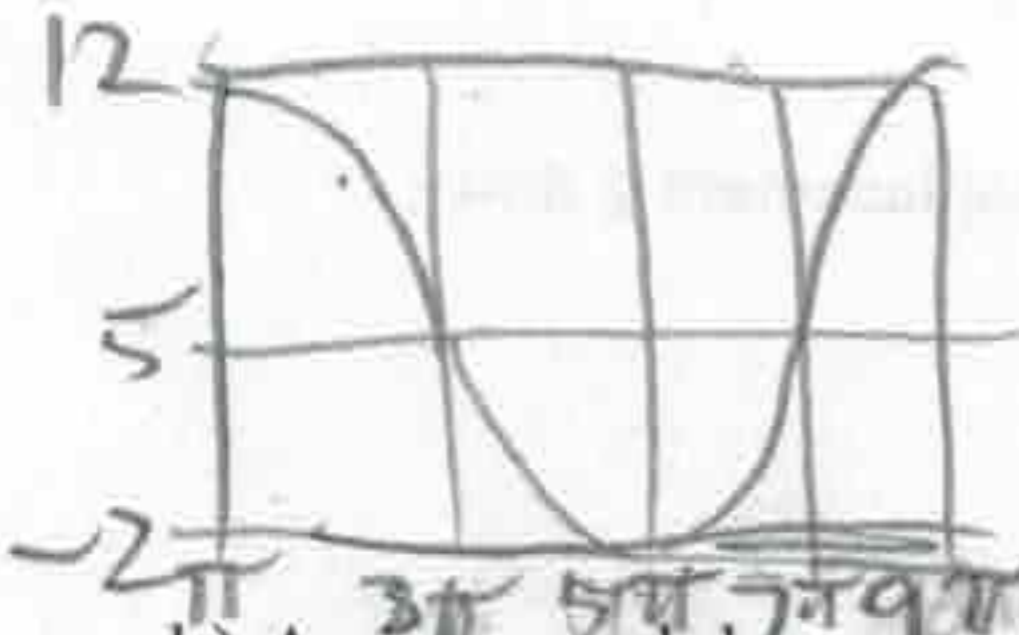
$$\{y \mid 3 \leq y \leq 13, y \in \mathbb{R}\}$$

b) State the **range** of the graph for the function  $y = a \sin\left(x + \frac{\pi}{6}\right) + b$ , where  $a > 0$  and  $b > 0$ .

$$\{y \mid (b-a) \leq y \leq (b+a), y \in \mathbb{R}\}$$

5a) A cosine graph has range  $-2 \leq y \leq 12$ , a maximum occurs at  $(\pi, 12)$ , and the period of the function is  $8\pi$ . What is a possible equation for this?

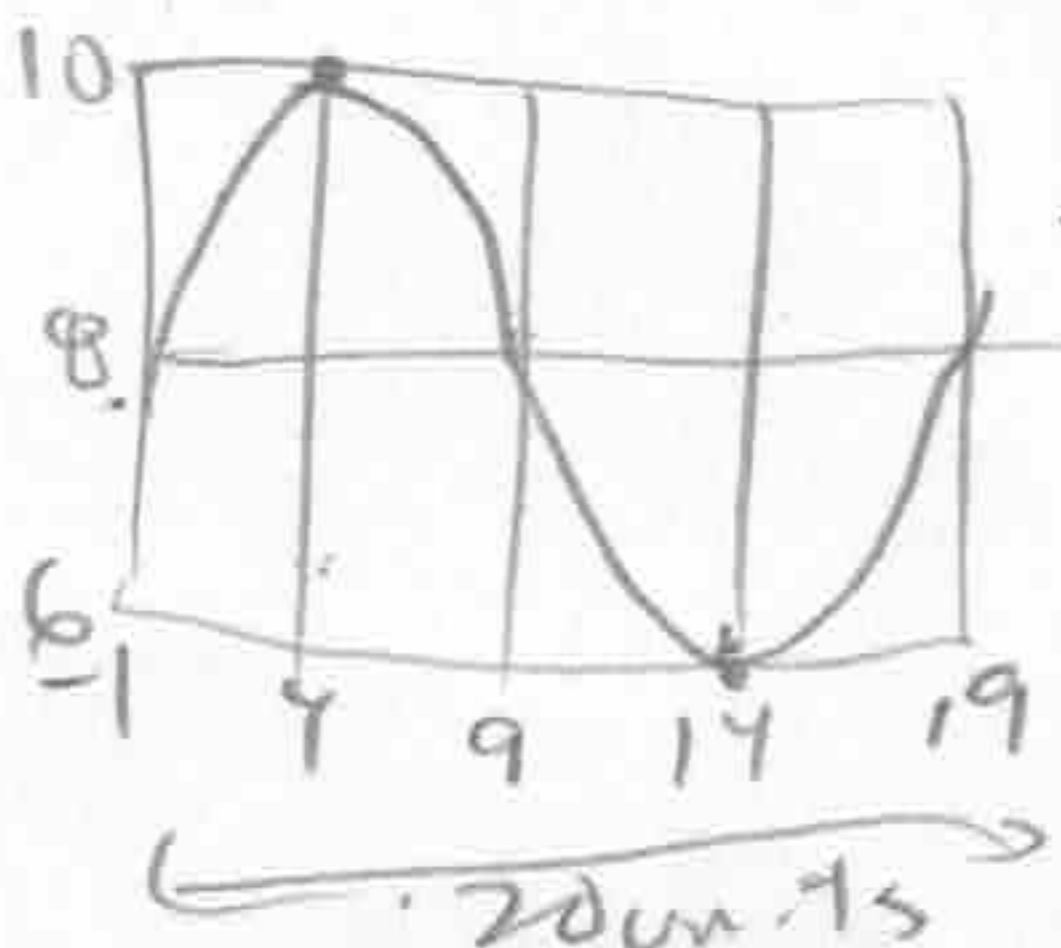
$$y = 7 \cos\left(\frac{1}{4}(x - \pi)\right) + 5$$



b) A sine graph has a maximum point at  $(4, 10)$  and the nearest minimum point to the right of this point is  $(14, 6)$ . What is a possible equation for this?

$$y = 2 \sin\left(\frac{2\pi}{20}(x - 4)\right) + 8$$

$$y = -2 \sin\left(\frac{2\pi}{20}(x - 14)\right) + 8$$





6. Consider this sinusoidal equation:  $y = 2 \cos\left(\frac{2\pi}{16}(x+9)\right) + 1$

a) Identify its key features:

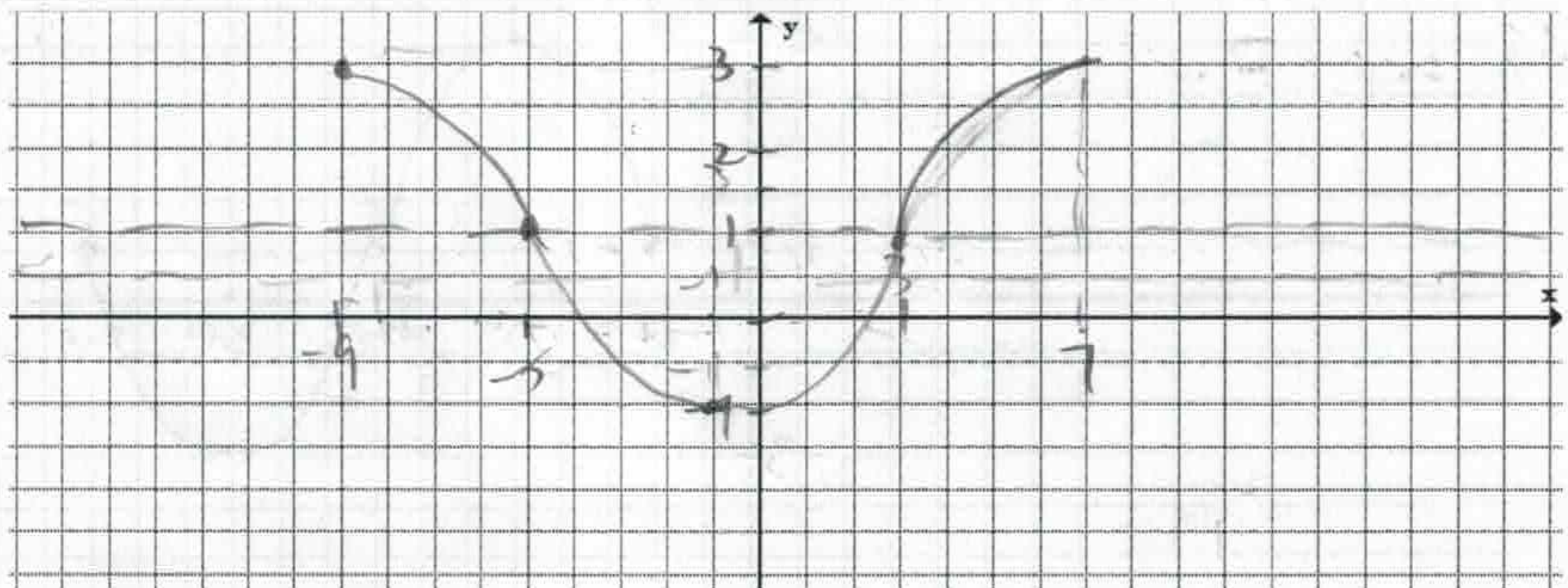
Sketch base graph's shape 	vertical displacement up 1	equation of center line $y = 1$
amplitude $1 + 2 = 3$	maximum $1 + 2 = 3$	minimum $1 - 2 = -1$
period 16 units	spacing $16 \div 4 = 4$	phase shift 9 left

b) Use either method shown in class to fill in the table with coordinates of 5 key points on the graph.

x	y
-9	3
-5	1
-1	-1
3	1
7	3

Know how to do this without using a graphing calculator. (You can use the calculator to check your results, though.)

c) Accurately sketch one cycle of the graph. Include the center line on your sketch.





$$2(x-30^\circ)$$

7. Consider this sinusoidal equation:  $y = 4\sin(2x - 60^\circ) + 1$

a) Identify its key features:

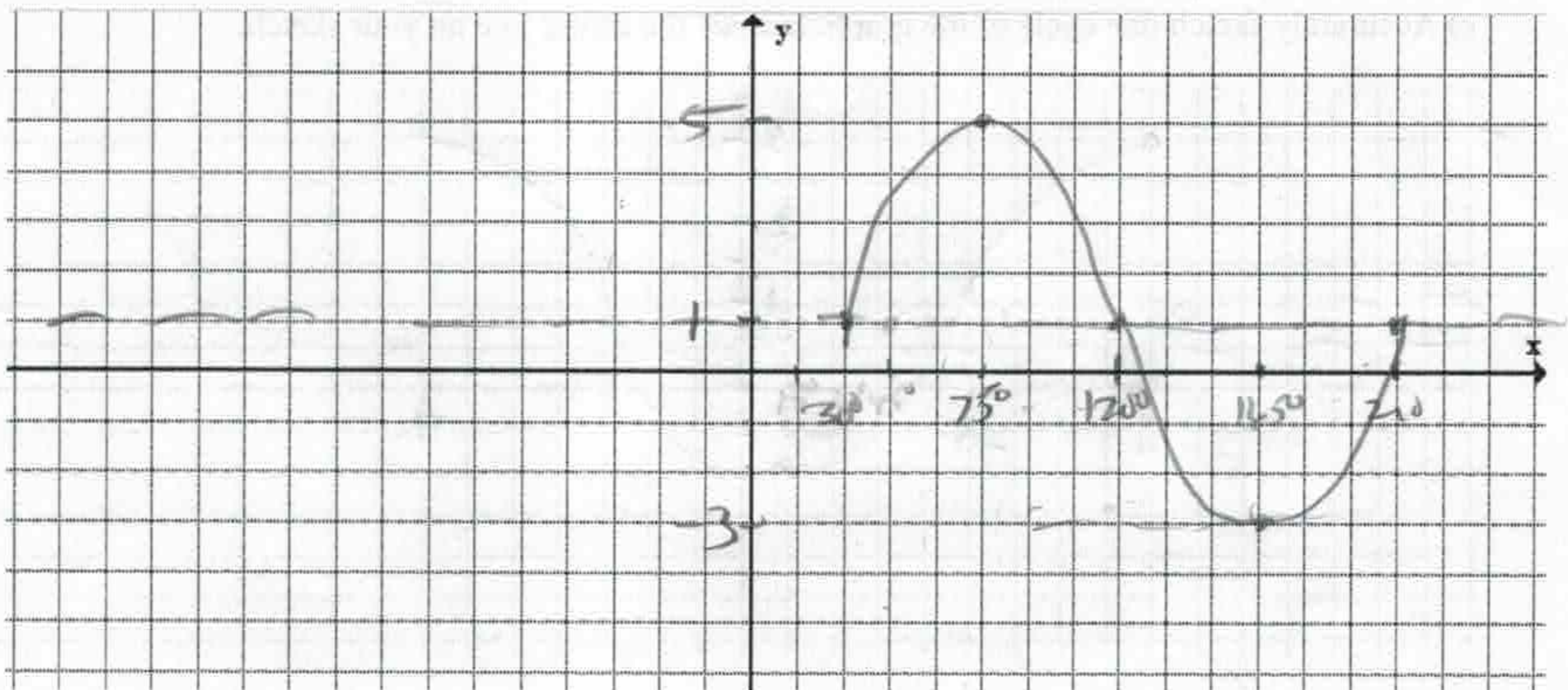
Sketch base graph's shape 	vertical displacement up 1	equation of center line $y = 1$
amplitude 4	maximum 5	minimum -3
period $\frac{360^\circ}{2} = 180^\circ$	spacing $\frac{180^\circ}{4} = 45^\circ$	phase shift 30° right.

b) Use either method shown in class to fill in the table with coordinates of 5 key points on the graph.

x	y
30°	1
75°	5
120°	1
165°	-3
210°	1

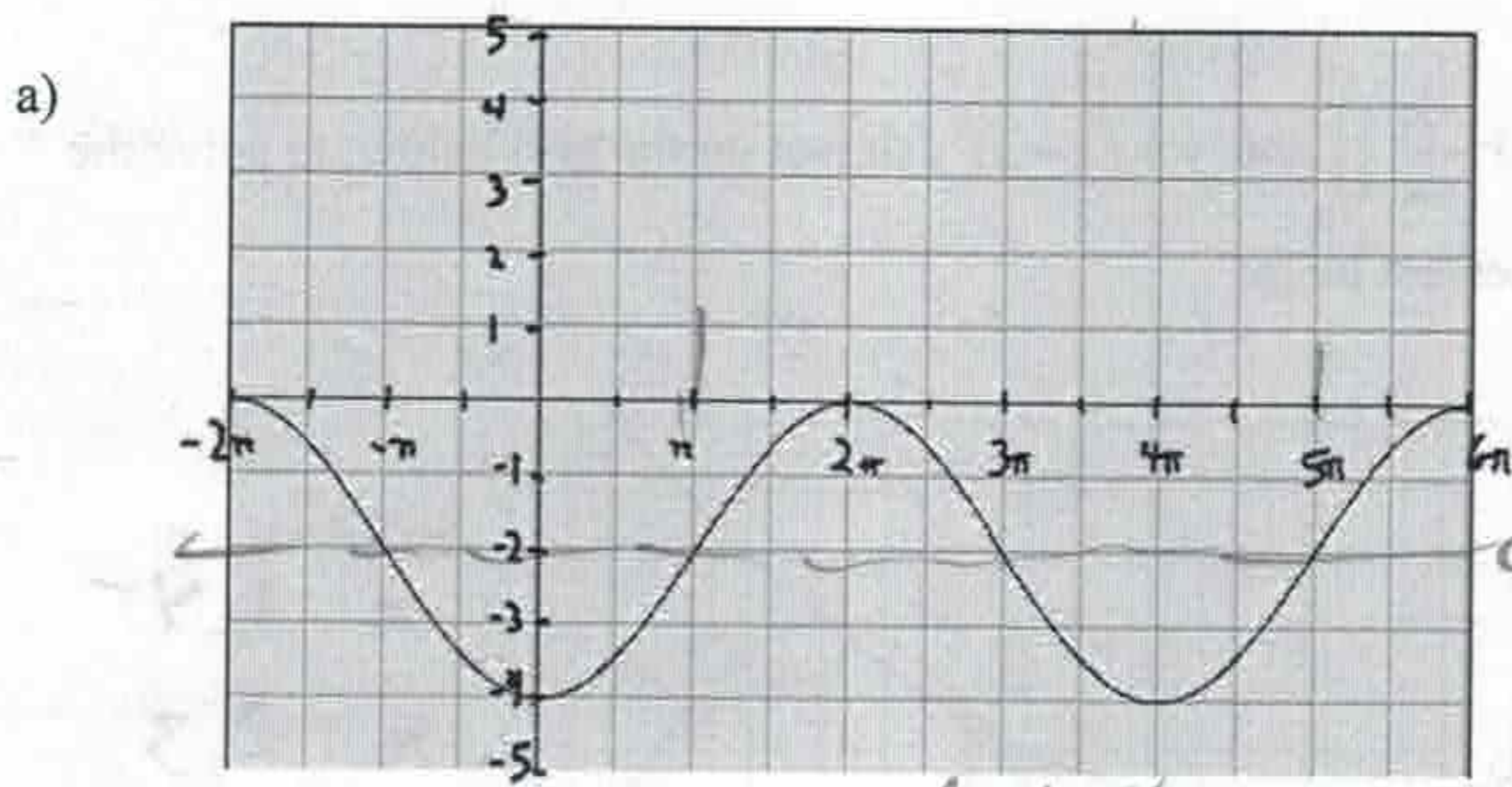
Know how to do this without using a graphing calculator. (You can use the calculator to check your results, though.)

c) Accurately sketch one cycle of the graph. Include the center line on your sketch.



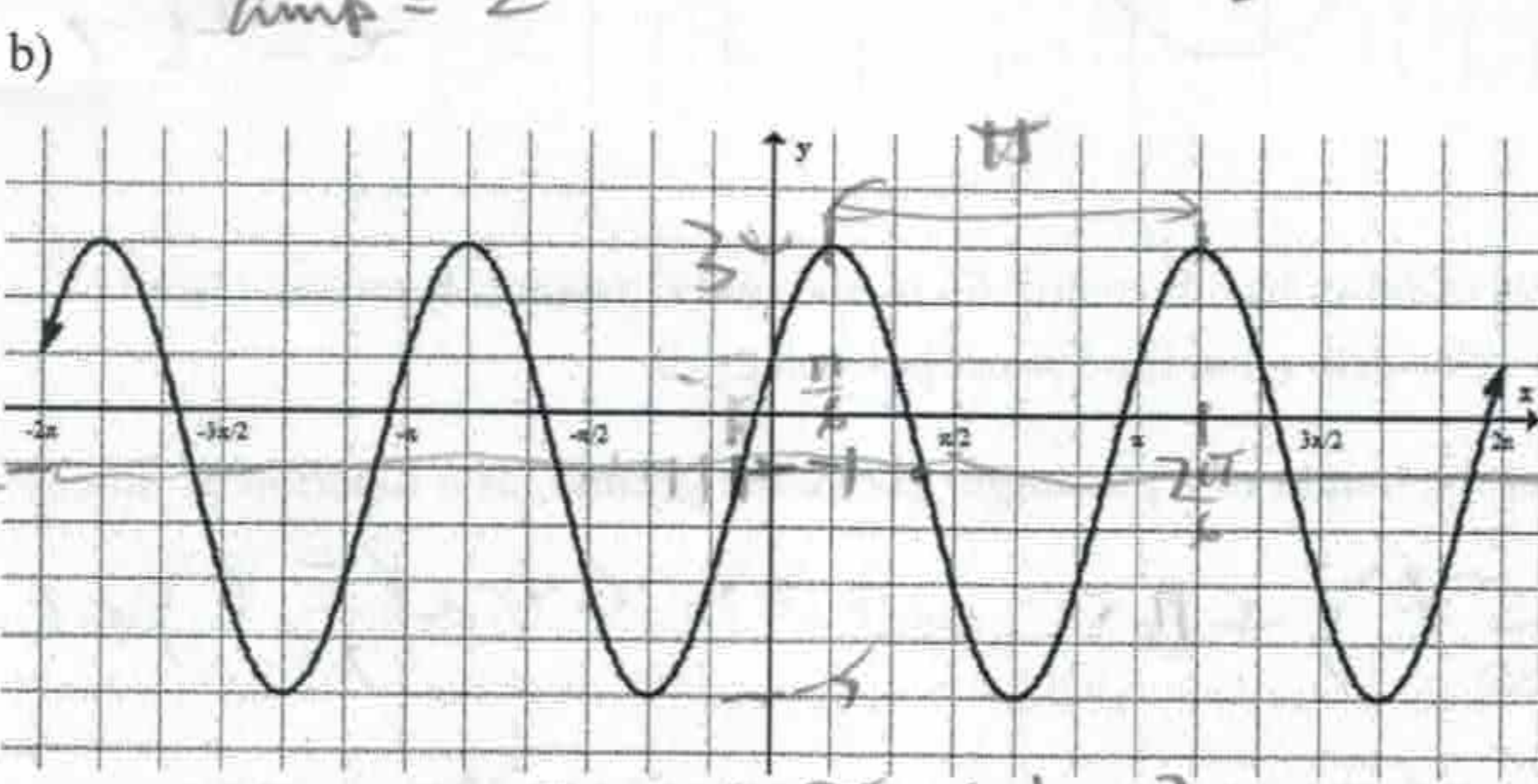


8. For each graph, find two different equations that create it. Give one equation using sine and a different one using cosine.



Possible sine equation:  
 $y = 2 \sin \frac{1}{2}(x - \pi) - 2$   
 Possible cosine equation:  
 $y = -2 \cos \frac{1}{2}x - 2$   
 or  $y = 2 \cos \frac{1}{2}(x + 2\pi) - 2$   
 $y = 2 \cos \frac{1}{2}(x - 2\pi) - 2$

C/L:  $y = -2$  period =  $4\pi$   
 amp = 2  $\therefore b = \frac{1}{2}$



Possible sine equation:  
 $y = 4 \sin 2(x + \frac{\pi}{2}) - 1$   
 Possible cosine equation:  
 $y = 4 \cos 2(x - \frac{\pi}{6}) - 1$

amp = 4 period =  $\pi$   $\therefore b = 2$

9. State the domain and period for each tangent equation:

a)  $y = \tan x$

b)  $y = \tan(5x)$

$\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in \mathbb{R}, n \in \mathbb{I}\}$   
 period =  $\pi$

$\{x \mid x \neq \frac{\pi}{10} + \frac{\pi}{5}n, x \in \mathbb{R}, n \in \mathbb{I}\}$   
 period =  $\frac{\pi}{5}$

10. The depth of water,  $h$  meters, at a certain ocean port, at time  $t$  hours, is given by the equation

$h(t) = 1.8 \sin \left( \frac{2\pi}{12.4}(t - 1.4) \right) + 2.6$ . (Note  $t = 0$  corresponds to midnight.)

a) What is the depth of the water at 7:00 A.M., correct to 2 decimal places?

$t = 7$ ;  $h(7) = 3.14 \text{ m}$

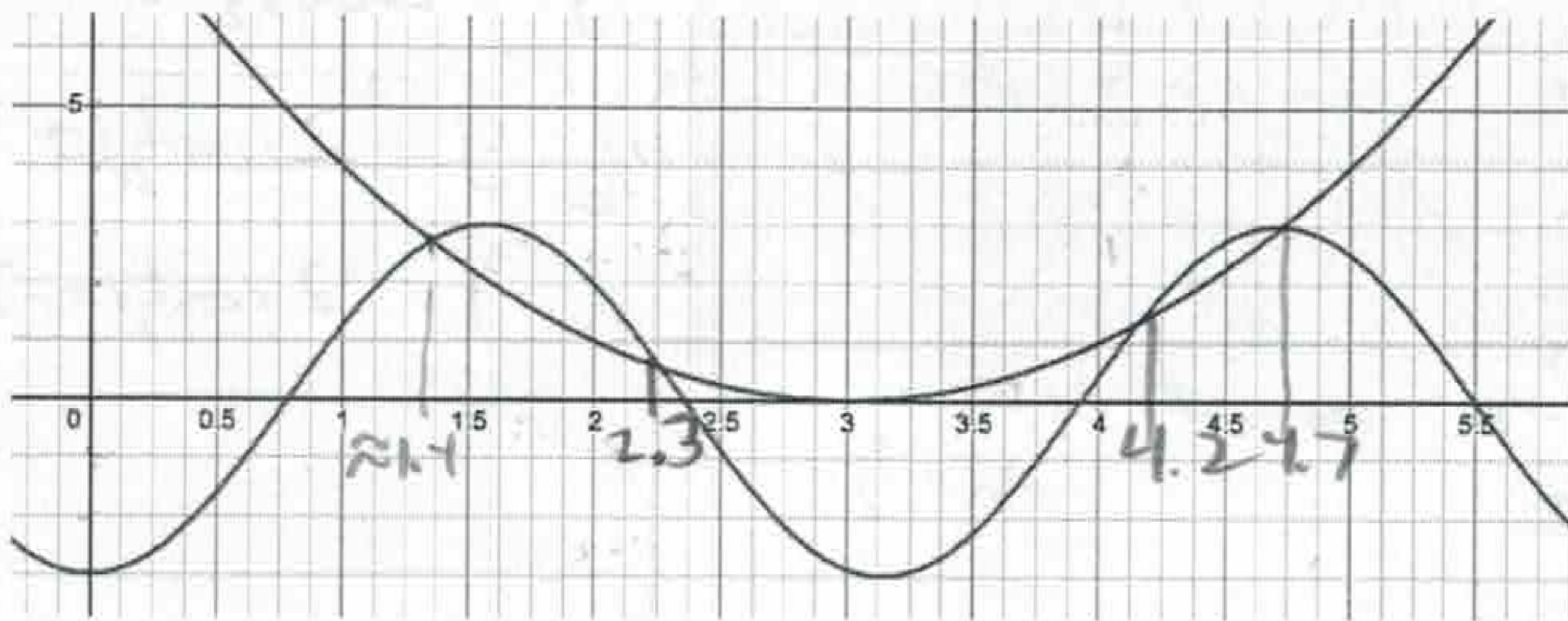
b) What is the depth of the water at 7:00 P.M., correct to 2 decimal places?

$t = 19$ ;  $h(19) = 3.17 \text{ m}$



11. We want to solve this equation *graphically*:  $2 \sin\left(2\left(x - \frac{\pi}{4}\right)\right) = (x-3)^2$

Use the graphs of  $y = 3 \sin\left(2\left(x - \frac{\pi}{4}\right)\right)$  and  $y = (x-3)^2$ , shown on the grid below, to solve the above equation, correct to the nearest tenth.



Solutions are:

$$x = 1.4$$

$$x = 2.3$$

$$x = 4.2$$

$$x = 4.7$$

12. A Ferris wheel with radius of 64 m has its center 65 m above the ground. It rotates once every 50 seconds. Suppose a rider gets on at the lowest point at  $t = 0$ .

a) Write an equation to model the height of a passenger above the ground, as a function of time.

$$h(t) = -64 \cos\left(\frac{2\pi}{50}t\right) + 65 \quad \text{or} \quad h(t) = -64 \cos\left(\frac{\pi}{25}t\right) + 65$$

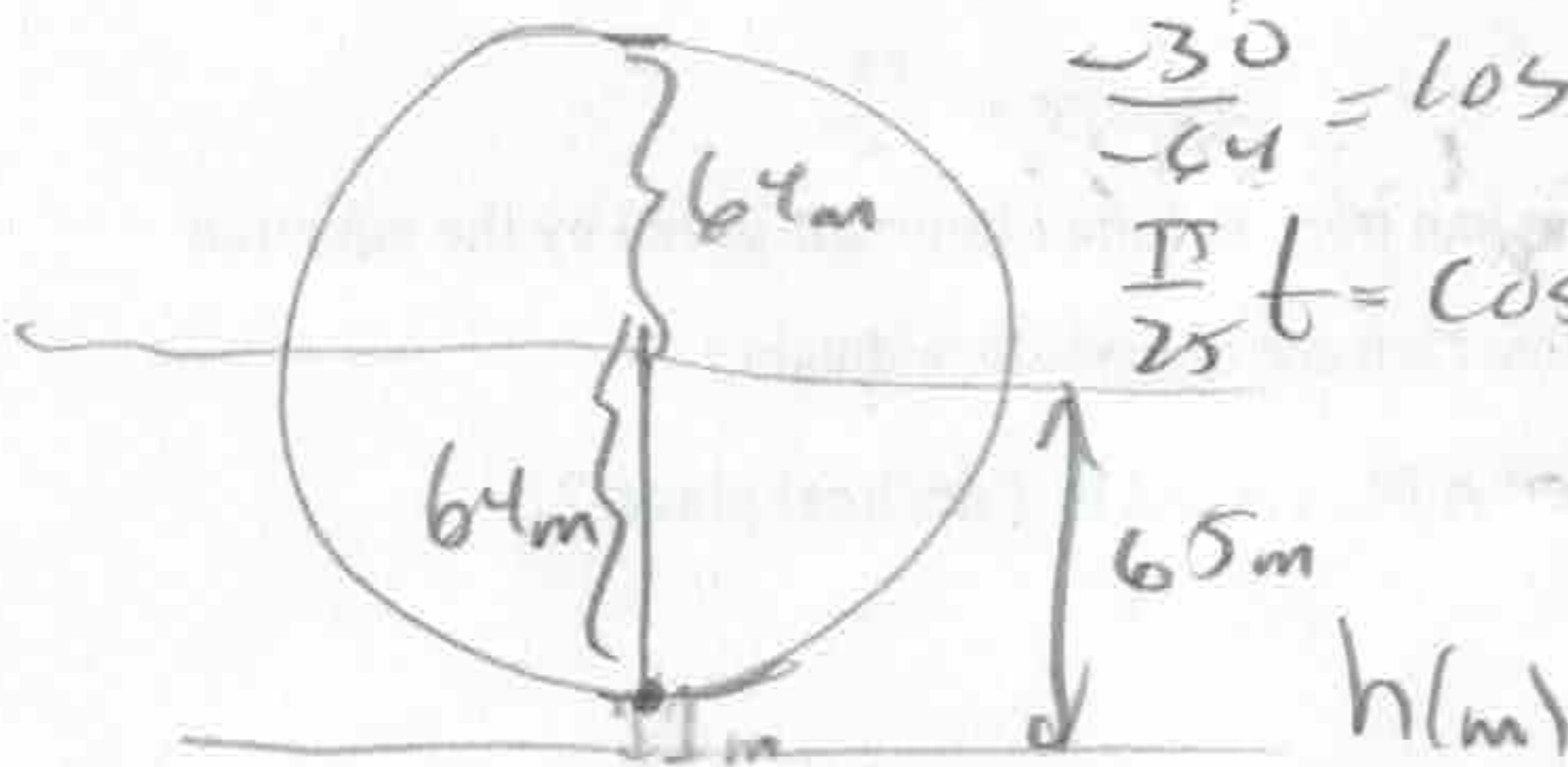
b) To the nearest tenth of a second, at what time will the passenger first be 35 m above the ground? Solve this algebraically. (If you like, check your answer by solving graphically.)

$$35 = -64 \cos\left(\frac{\pi}{25}t\right) + 65$$

$$\frac{-30}{-64} = \cos\left(\frac{\pi}{25}t\right)$$

$$\frac{\pi}{25}t = \cos^{-1}\left(\frac{-30}{-64}\right) = 1.0822$$

$$t = \frac{25(1.0822)}{\pi} = \underline{\underline{8.65}}$$



$$\text{Amp} = 64$$

