Chapter 8 Hand-in Assignment – Logarithms

Name: _____

1. Describe how the graph of each logarithmic function can be obtained from the graph of $y = \log_2 x$.

a) $y = \log_2\left(-5(x+4)\right) - 1$

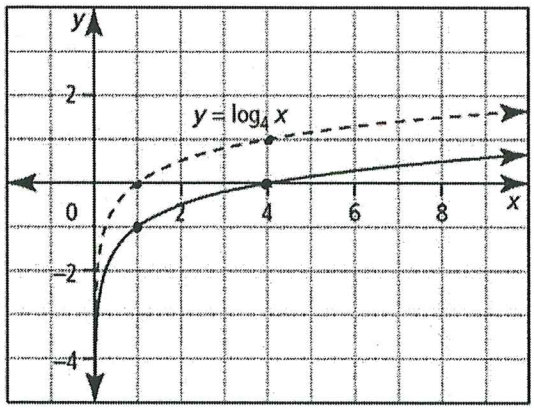
- reflect over x-axis
- HC by 5
- 4 to left, 1 down

b) $y = 2\log_2 (x-7) + 4$

- VE by 2
- 7 to right, 4 up

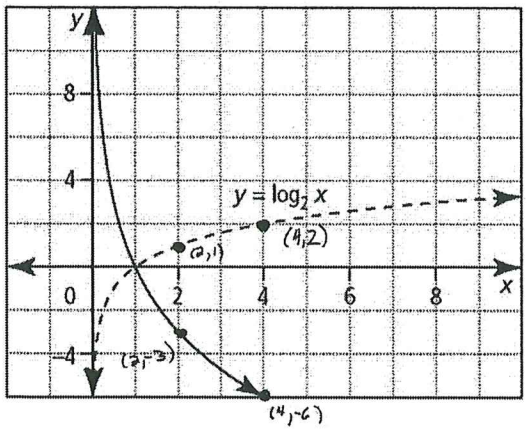
2. For each graph, the solid curve is a transformation of the dashed curve. Write the equation of each solid graph.

a)



$y = \log_4 x - 1$

b)



$y = -3\log_2 x$

3. Given the base function $y = \log_2 x$ and its transformed function, $y = -2\log_2(x+4) - 1$ do the following:

- List the transformations that will occur and give the mapping notation

• reflect over x-axis
• VE by 2
• 4 to left
• 1 down

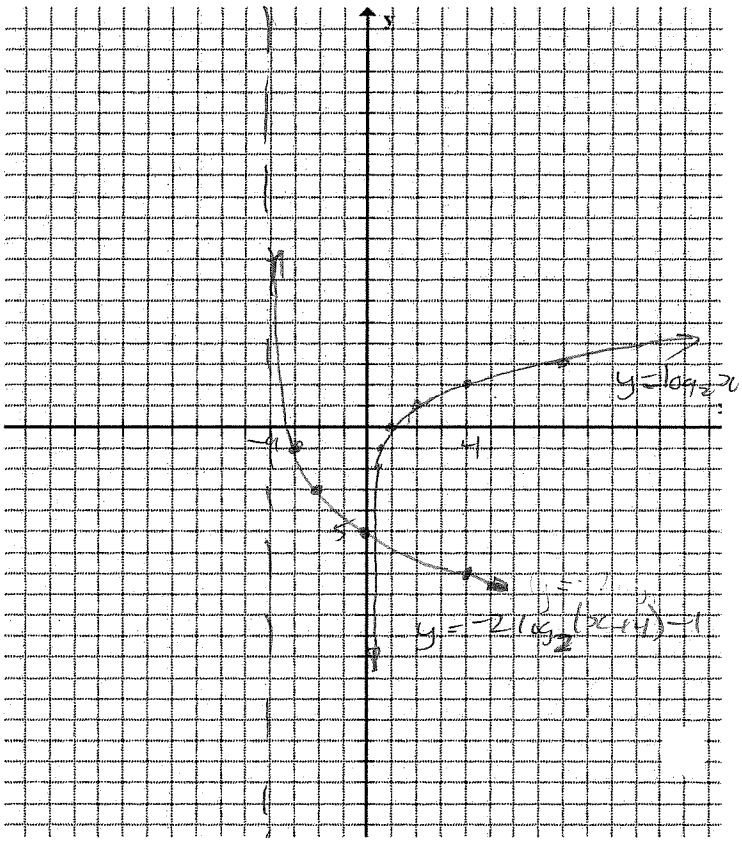
$(x, y) \rightarrow (x-4, -2y-1)$

- Complete the tables, showing 5 key points for each table. Include mapping notation on the table heading.

original

x	y
1/4	−2
1/2	−1
1	0
2	1
4	2
8	3

x−4	−2y−1
−3 3/4	3
−3 1/2	1
−3	−1
−2	−3
0	−5
4	−7



- On the provided grid, sketch and label the original and transformed graphs.

- For the transformed function, give
  - Domain $\{x \mid x > -4, x \in \mathbb{R}\}$
  - Range $\{y \mid y \in \mathbb{R}\}$
  - The equation of its asymptote $x = -4$
  - The coordinates of its x-intercept and y-intercept

x-int $(y = 0)$

$0 = -2\log_2(x+4) - 1$

$1 = -2\log_2(x+4)$

$-\frac{1}{2} = \log_2(x+4)$

$2^{-1/2} = x+4$

$x = 2^{-1/2} - 4 = \frac{1}{\sqrt{2}} - 4$

$\doteq -3.29 \therefore (-3.29, 0)$

y-int $(x = 0)$

$(0, -5)$

4. The graph of $y = \log_3 x$ has been transformed as described below. Give the equation of each transformed function: $y = a\log_c[b(x-h)]+k$

a) $y = \log_3 x$ is expanded vertically by a factor of 4, expanded horizontally by a factor of 3, reflected across the y-axis, and is translated 2 left and 5 down.

$a=4$ $h=-2$ $b=-\frac{1}{3}$ $k=-5$ $c=3$ $y = 4\log_3\left[-\frac{1}{3}(x+2)\right]-5$

b) $y = \log_3 x$ is compressed vertically by a factor of ½, expanded horizontally by a factor of 8, reflected across the x-axis, and is translated 3 right and 6 up.

$a=-\frac{1}{2}$ $h=3$ $b=\frac{1}{8}$ $k=6$ $c=3$ $y = -\frac{1}{2}\log_3\left[\frac{1}{8}(x-3)\right]+6$

5. Write in logarithmic form.

a) $4^{-2} = 0.0625$ $\log_4 0.0625 = -2$

b) $5^3 = r+6$ $\log_5(r+6) = 3$

c) $e^x = 8$ $\ln 8 = x$

6. Write in exponential form.

a) $\log_2 512 = 9$ $2^9 = 512$

b) $\ln(16) = t$ $e^t = 16$

c) $\log_2(a-4) = b$ $2^b = a-4$

7. Use the definition of logarithms to find the value of each expression below.

a) $\log_3 81 = \log_3(3^4) = 4$

b) $\log_4\left(\frac{1}{64}\right) = \log_4\left(\frac{1}{4^3}\right) = \log_4 4^{-3} = -3$

8. Solve the following equations for x. If answer is not exact, give it correct to **2 decimal places.**

a) $\log_4(x-8) = 5$ $4^5 = x-8$ $1024 = x-8$ $x = 1024+8 = 1032$

b) $\log_x(18) = \frac{3}{4}$ $x^{3/4} = 18$ $(x^{3/4})^{4/3} = (18)^{4/3}$ $x = \sqrt[3]{18^4} = 47.17$

c) $\ln(x)+\ln(8) = \ln 32$ $8x = 32$ $x = \frac{32}{8} = 4$

d) $\log_6(3x^7)-\log_6(x^6) = 2$ $\frac{3x^7}{x^6} = 6^2$ $3x = 36$ $x = \frac{36}{3} = 12$

Common Logs

9. Evaluate each of the following on your calculator. Give answers correct to **4 decimal places**.

a) $\log_7 18$

$= \frac{\log 18}{\log 7}$

$= 1.4854$

b) $4\log_3 29$

$= 4\left(\frac{\log 29}{\log 3}\right)$

$= 12.2602$

c) $2\ln 53$

$= 7.9406$

10. Evaluate each of the following.

a) $\log_8 8 = 1$

b) $\log_{16} 1 = 0$

c) $\log_3 3^5 = 5$

d) $7^{\log_7 13} = 13$

e) $\ln e^3 = 3$

f) $\ln(1) = 0$

11. Use logarithm laws to expand the following logarithms completely.

a) $\log_2\left(\frac{8x^4\sqrt{y}}{w^3}\right)$

$= \log_2 8 + \log_2 x^4 + \log_2 y^{\frac{1}{2}} - \log w^3$

$= 3 + 4\log_2 x + \frac{1}{2}\log_2 y - 3\log_2 w$

b) $\log_3\left(\frac{\sqrt[4]{x}}{yz^2}\right)$

$= \log_3 x^{1/4} - \log_3 y - \log_3 z^2$

$= \frac{1}{4}\log_3 x - \log_3 y - 2\log_3 z$

12. Use logarithm laws to condense each expression into a single logarithm.

a) $3\log 2x - 5\log x - \log 2 + 8\log x$

$= \log\left[\frac{(2x)^3(x^8)}{x^5(2)}\right]$

$= \log\left(\frac{8x^3 \cdot x^8}{2x^5}\right)$

$= \log(x^6)$

b) $\frac{\log(x-7)}{\log 5}$

NOTE $\neq \log\left(\frac{x-7}{5}\right)$

$= \log_5(x-7)$ { change of base rule

13. Determine the value of $\log_2\left(16a^3b^2\right)$ if $\log_2 a = 5$ and $\log_2 b = 4$.

$= \log_2(16) + \log_2(a^3) + \log_2 b^2$

$= 4 + 3(5) + 2(4)$

$= 4 + 15 + 8 = 27$

14. Solve each equation for x. Reject extraneous solutions. Give answers correct to **2 decimal places.**

a) $\log_4(2x+1)-\log_4 3=\log_4 11$

$\log_4\left(\frac{2x+1}{3}\right)=\log_4 11$

$\frac{2x+1}{3}=11$

$2x+1=33$

$2x=32$

$x=16$

b) $\log_2 x=3-\log_2(x+2)$

$\log_2 x=\log_2 2^3-\log_2(x+2)$

$\log_2 x=\log_2\left(\frac{8}{x+2}\right)$

$x=\frac{8}{x+2}$

$x(x+2)=8$

$x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0$ $\quad x-2=0$

$x=-4$ $\quad x=2$ ✓

reject

x must be >0

c) $\log_5(3x+1)+\log_5(x-3)=3$

$\log_5[(3x+1)(x-3)]=\log_5 5^3$

$(3x+1)(x-3)=125$

$3x^2-9x+x-3=125$

$3x^2-8x-128=0$

$a \quad b \quad c$

$ac=3(-128)=-384=-24\times 16$

$b=-8=-24+16$

$3x^2-24x+16x-128=0$

$3x(x-8)+16(x-8)=0$

$(3x+16)(x-8)=0$

$3x+16=0$ $\quad x-8=0$

$x=-\frac{16}{3}$ $\quad x=8$

reject

15. Solve each equation for x. Give answers correct to **2 decimal places.**

a) $e^{x+3} = 45$

$\ln(e^{x+3}) = \ln(45)$

$(x+3)\ln e = \ln 45$ ($\ln e = 1$)

$x+3 = \ln 45$

$x = (\ln 45) - 3 = 3.80666249 - 3 \doteq 0.81$

b) $2^{x+5} = 5^{3x-4}$

$(x+5)\log 2 = (3x-4)\log 5$

$x\log 2 + 5\log 2 = 3x\log 5 - 4\log 5$

$x\log 2 - 3x\log 5 = -4\log 5 - 5\log 2$

$x(\log 2 - 3\log 5) = -4\log 5 - 5\log 2$

$\rightarrow x = \dfrac{(-4\log 5 - 5\log 2)}{(\log 2 - 3\log 5)}$

$x \doteq 2.39$

c) $6^{2x+5} = 3(7^{8-x})$

$(2x+5)\log 6 = \log 3 + (8-x)\log 7$

$2x\log 6 + 5\log 6 = \log 3 + 8\log 7 - x\log 7$

$2x\log 6 + x\log 7 = \log 3 + 8\log 7 - 5\log 6$

$x(2\log 6 + \log 7) = (\log 3 + 8\log 7 - 5\log 6)$

$x = \dfrac{[\log 3 + 8\log 7 - 5\log 6)}{(2\log 6 + \log 7)} \doteq 1.39$

16. An investment of \$2000 pays interest at a rate of 1.5% per year. Determine how long it takes for the investment to grow to \$3000, if the interest is compounded quarterly. **Solve algebraically, not graphically.**

0.015

$3000 = 2000\left(1 + \dfrac{0.015}{4}\right)^{4t}$ $\quad t$ = time in years

$\dfrac{3000}{2000} = (1.00375)^{4t}$

$1.5 = (1.00375)^{4t}$

$\log 1.5 = 4t \log 1.00375$

$t = \dfrac{\log 1.5}{4\log 1.00375} = 27.0815\ldots \rightarrow 27\frac{1}{4}$ years

27 years would not be long enough!

17. A type of bacteria doubles every 3 days. How long would it take a sample of 400 cells to grow to 9,000 cells?

$9000 = 400(2)^{t/3}$

$\frac{9000}{400} = 2^{t/3}$

$22.5 = 2^{t/3}$

$\log 22.5 = \frac{t}{3}\log 2$

$t = \frac{3\log 22.5}{\log 2} = 13.4755 \approx 13.5 \text{ days}$

18. A sample of a radioactive substance decays from 400 mg to 80 mg in 76 days. What is the half-life of this substance? ***Solve algebraically.*** Express your answer correct to two decimal places.

$80 = 400\left(\frac{1}{2}\right)^{76/t_{\frac{1}{2}}}$ $\quad t_{\frac{1}{2}} = \text{half-life}$

$\frac{80}{400} = \left(\frac{1}{2}\right)^{76/t_{\frac{1}{2}}}$

$0.2 = \left(\frac{1}{2}\right)^{76/t_{\frac{1}{2}}}$

$\log 0.2 = \left(\frac{76}{t_{\frac{1}{2}}}\right)\log\frac{1}{2}$

$t_{\frac{1}{2}} = \frac{76\log\frac{1}{2}}{\log 0.2} = 32.7314\ldots$

$\approx 32.73 \text{ days}$

19. Suppose that solution A is 500 times as acidic as than solution B. If we know the pH of solution B is 9.1, what is the pH of solution A?

$I = I_0(10)^{(p_B - p_A)}$

$\frac{I}{I_0} = 500 = 10^{(9.1 - p_A)}$

$\log 500 = \log\left(10^{(9.1 - p_A)}\right)$

$\log 500 = 9.1 - p_A$

$p_A = 9.1 - \log 500 = 6.4$

20. An earthquake in California measured 4.8 on the Richter scale. Another earthquake near Japan was 300 times more intense. What was the Richter scale reading for the earthquake near Japan?

$\frac{I}{I_0} = 300 = 10^{R-4.8}$

$\log 300 = (R-4.8)\log 10$

$R = (\log 300) + 4.8$

$\approx \boxed{7.3}$

21. The loudness level of a vacuum cleaner is 75 dB and that of a chainsaw is 110 dB. How many times as loud as a vacuum cleaner is a chainsaw?

$\frac{I}{I_0} = 10^{\frac{110-75}{10}} = 10^{35/10} = 10^{3.5}$

$\approx 3162.2\ldots$

$\approx \boxed{3162}$ times as loud

22. The population of Toronto is given by $P(t) = 4{,}000{,}000e^{0.012t}$, where $t = 0$ corresponds to the year 2000. In what year will the population reach 5,800,000?

$5\,800\,000 = 4\,000\,000e^{0.012t}$

$\frac{5\,800\,000}{4\,000\,000} = e^{0.012t}$

$1.45 = e^{0.012t}$

$\ln 1.45 = \ln(e^{0.012t})$

$\ln 1.45 = 0.012t$

$t = \frac{\ln 1.45}{0.012} \approx 30.9634\ldots$

≈ 31 years

$\therefore$ In 2000 + 31

$= \boxed{2031}$