C_21 Key and More Solving Practice

Wednesday, November 13, 2019 4:49 PM

C_21 More Solving Practice with Solutions

Practice Solving Logarithmic & Exponential Equations

1. Solve each equation for x.

a) $6^{3x-6} = 1$

b) $4^{8x} = \frac{1}{16}$

c) $x^{4/5} = 23$

d) $3^{x} = 125$

e) $65 = e^{7x}$ (e is a number, just like π is a number)

f) $7(2^{x}) = 5^{x-2}$

g) $17^{x+4} = 196^{3x-2}$

2. Solve these logarithmic equations for x.

a) $\log_3(4x-1) = 2$

b) $\log_5 24 - \log_5 2 = \log_5 3x$

c) $\log(8+2x) = \log(7x-2)$

d) $\log_{2x} 64 = 2$

e) $\log_x 125 = 3$

f) $\log x + \log 12 = \log 8$

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3. Solve these logarithmic equations for x.

a) $x\log 26 = \log 13$

b) $\log(5x+4) = 3$

c) $\log_4 188 = x$

d) $\log 42 = \log 14 - \log x$

e) $\ln x - \ln 4 = \ln 5$ ("ln" means $\log_e$)

f) $\ln x - \ln 4 = 5$ (This is NOT the same question as part e)

g) $\log_2(x^2+8) - \log_2 6 = \log_2 x$

h) $\log_5(3x+1) + \log_5(x-3) = 3$

i) $\log_2(x-2) + \log_2 x = \log_2 3$

j) $\log_5(x-6) = 1 - \log_5(x-2)$

k) $2\log_3 x - \log_3(x+3) - 3 = 0$

l) $\log_5(x+1) + \log_5(x-3) = 1$

Solutions

1. Solve each equation for x.

a) $6^{3x-6} = 1$

$6^{3x-6} = 6^0$

$\Rightarrow 3x - 6 = 0$

$3x = 6$

$\boxed{x = 2}$

OR

$\log 6^{3x-6} = \log 1$

$(3x-6)\log 6 = \log 1$

$3x\log 6 - 6\log 6 = \log 1$

$3x\log 6 = \log 1 + 6\log 6$

$x(3\log 6) = \log 1 + 6\log 6$

$x = \dfrac{\log 1 + 6\log 6}{(3\log 6)} = \boxed{2}$

b) $4^{8x} = \dfrac{1}{16}$

$4^{8x} = \dfrac{1}{4^2}$

$4^{8x} = 4^{-2}$

$\Rightarrow 8x = -2$

$x = -2/8$

$\boxed{x = -1/4}$

OR

$\log 4^{8x} = \log 1/16$

$8x \log 4 = \log 1/16$

$x(8\log 4) = \log 1/16$

$x = \dfrac{\log 1/16}{(8\log 4)}$

$\boxed{x = -1/4}$

c) $x^{4/5} = 23$

$\left(x^{4/5}\right)^{5/4} = (23)^{5/4}$

$\boxed{x \doteq 50.37}$

d) $3^x = 125$

$\log 3^x = \log 125$

$x\log 3 = \log 125$

$x = \dfrac{\log 125}{\log 3}$

$\boxed{x \doteq 4.39}$

OR

Change form:

$\log_3 125 = x$

$x = \dfrac{\log 125}{\log 3}$

(change of Base Law)

$\boxed{x \doteq 4.39}$

e) $65 = e^{7x}$ (e is a number, just like π is a number) OR

On a TI-83, it's found by doing $\boxed{2^{nd}}$ $\boxed{\div}$

$$\log 65 = \log e^{7x}$$
$$\log 65 = 7x \log e$$
$$\log 65 = x(7\log e)$$
$$x = \frac{\log 65}{(7\log e)} \doteq \boxed{0.60}$$

Change form

$$\log_e 65 = 7x$$
$$\frac{\log 65}{\log e} = 7x$$
$$x = \frac{\left(\frac{\log 65}{\log e}\right)}{7} \doteq \boxed{0.60}$$

By the way, $\log_e 65 = LN\ 65$, so a faster way to evaluate is $x = \frac{LN\ 65}{7}$, $x \doteq 0.60$

f) $7(2^x) = 5^{x-2}$

$$\log\left[7(2^x)\right] = \log\left(5^{x-2}\right)$$
$$\log 7 + \log 2^x = (x-2)\log 5$$
$$\log 7 + x\log 2 = x\log 5 - 2\log 5$$
$$x\log 2 - x\log 5 = -2\log 5 - \log 7$$
$$x(\log 2 - \log 5) = -2\log 5 - \log 7$$
$$x = \frac{(-2\log 5 - \log 7)}{(\log 2 - \log 5)} \doteq \boxed{5.64}$$

g) $17^{x+4} = 196^{3x-2}$

$$\log 17^{x+4} = \log 196^{3x-2}$$
$$(x+4)\log 17 = (3x-2)\log 196$$
$$x\log 17 + 4\log 17 = 3x\log 196 - 2\log 196$$
$$x\log 17 - 3x\log 196 = -2\log 196 - 4\log 17$$
$$x(\log 17 - 3\log 196) = -2\log 196 - 4\log 17$$
$$x = \frac{(-2\log 196 - 4\log 17)}{(\log 17 - 3\log 196)} \doteq \boxed{1.68}$$

$(\log 17 - 3\log 196)$

2. Solve these logarithmic equations for x.

a) $\log_3(4x-1)=2$

$$3^2 = 4x-1$$
$$9 = 4x-1$$
$$\frac{10}{4} = \frac{4x}{4} \qquad x = \frac{10}{4} = \boxed{\frac{5}{2}}$$

b) $\log_5 24 - \log_5 2 = \log_5 3x$

$$\log_5\left(\frac{24}{2}\right) = \log_5 3x$$
$$\log_5(12) = \log_5 3x$$
$$\Rightarrow \frac{12}{3} = \frac{3x}{3} \qquad \boxed{x=4}$$

c) $\log(8+2x)=\log(7x-2)$

$$\Rightarrow 8+2x = 7x-2$$
$$\frac{-5x}{-5} = \frac{-10}{-5}$$
$$\boxed{x=2}$$

d) $\log_{2x} 64 = 2$

$$(2x)^2 = 64$$
$$\frac{4x^2}{4} = \frac{64}{4}$$
$$x^2 = 16$$

$x = \pm 4$, but only $\boxed{x=4}$ is valid, since base must be > 0.

e) $\log_x 125 = 3$

$$\sqrt[3]{x^3} = \sqrt[3]{125}$$

$$\boxed{x = 5}$$

f) $\log x + \log 12 = \log 8$

$$\log(12x) = \log 8$$

$$\Rightarrow \frac{12x}{12} = \frac{8}{12}$$

$$x = \frac{8}{12}, \quad \boxed{x = \frac{2}{3}}$$

3. Solve these logarithmic equations for x.

a) $x \log 26 = \log 13$

$$x = \frac{\log 13}{\log 26}$$

$$\boxed{x \doteq 0.79}$$

b) $\log(5x+4) = 3$

$$10^3 = 5x+4$$

$$1000 = 5x+4$$

$$996 = 5x$$

$$x = \frac{996}{5}$$

$$\boxed{x = 199.2}$$

c) $\log_4 188 = x$

$$x = \frac{\log 188}{\log 4}$$ (Change of base law)

$$\boxed{x \doteq 3.78}$$

d) $\log 42 = \log 14 - \log x$

$\log 42 = \log\left(\frac{14}{x}\right)$

$\Rightarrow \quad 42 = \frac{14}{x}$

$\frac{42x}{42} = \frac{14}{42}$

$x = \frac{14}{42}$

$\boxed{x = 1/3}$

e) $\ln x - \ln 4 = \ln 5$ ("ln" means $\log_e$)

$\ln\left(\frac{x}{4}\right) = \ln 5$

$\Rightarrow \quad \frac{x}{4} = 5$

$\boxed{x = 20}$

f) $\ln x - \ln 4 = 5$ (This is NOT the same question as part e)

$\ln\left(\frac{x}{4}\right) = 5$

$e^5 = \frac{x}{4}$

$x = 4e^5$,

$\boxed{x \doteq 593.65}$

g) $\log_2\left(x^2+8\right) - \log_2 6 = \log_2 x$

$\log_2\left(\frac{x^2+8}{6}\right) = \log_2 x$

$\Rightarrow \quad \frac{x^2+8}{6} = x$

$x^2+8 = 6x$

$x^2-6x+8 = 0$

$(x-2)(x-4) = 0$

$\boxed{x = 2, \quad x = 4}$

Both are valid answers.

h) $\log_5(3x+1)+\log_5(x-3)=3$

$\log_5\left[(3x+1)(x-3)\right]=3$

$\log_5(3x^2-9x+x-3)=3$

$5^3=3x^2-8x-3$

$0=3x^2-8x-3-125$

$3x^2-8x-128=0$

$x=\dfrac{8\pm\sqrt{64-(4)(3)(-128)}}{2(3)}$

$x=\dfrac{8\pm\sqrt{64+1536}}{6}$

$x=\dfrac{8\pm\sqrt{1600}}{6}$

$x=\dfrac{8\pm 40}{6}$

$\to x=\dfrac{48}{6}=\boxed{8}$

$\to x=\dfrac{32}{6}=\cancel{+\dfrac{16}{3}}$

reject, extraneous root

i) $\log_2(x-2)+\log_2 x=\log_2 3$

$\log_2\left[(x-2)(x)\right]=\log_2 3$

$\log_2(x^2-2x)=\log_2 3$

$\Rightarrow x^2-2x=3$

$x^2-2x-3=0$

$(x+1)(x-3)=0$

$\cancel{x=-1}$ reject, extraneous

$\boxed{x=3}$ valid

j) $\log_5(x-6)=1-\log_5(x-2)$

$\log_5(x-6)+\log_5(x-2)=1$

$\log_5\left[(x-6)(x-2)\right]=1$

$\log_5(x^2-2x-6x+12)=1$

$5^1=x^2-8x+12$

$0=x^2-8x+7$

$0=(x-1)(x-7)$

$\cancel{x=1}$ reject

$\boxed{x=7}$ valid

k) $2\log_3 x - \log_3(x+3) - 3 = 0$

$$\log_3 x^2 - \log_3 (x+3) = 3$$

$$\log_3 \left(\frac{x^2}{x+3}\right) = 3$$

$$3^3 = \frac{x^2}{x+3}$$

$$27 = \frac{x^2}{x+3}$$

$$27(x+3) = x^2$$

$$27x + 81 = x^2$$

$$0 = x^2 - 27x - 81$$

$$x = \frac{27 \pm \sqrt{(-27)^2 - (4)(1)(-81)}}{2(1)}$$

$$x = \frac{27 \pm \sqrt{1053}}{2}$$

$$\boxed{29.72}$$

$$\cancel{-2.72}$$

l) $\log_5(x+1) + \log_5(x-3) = 1$

$$\log_5 \left[(x+1)(x-3)\right] = 1$$

$$\log_5 (x^2 - 3x + 1x - 3) = 1$$

$$5^1 = x^2 - 2x - 3$$

$$0 = x^2 - 2x - 8$$

$$0 = (x+2)(x-4)$$

$$\cancel{x = -2} \qquad \boxed{x = 4}$$