C_15 Key correct 2022 Ch 6 Hand-in

Tuesday, February 22, 2022   4:00 PM

C_15 Ch 6 Hand-in 2022

Chapter 6 Hand-in Assignment – Trigonometric Identities

Name:

1. Simplify each expression.

a) $\dfrac{\tan x}{\sec x}$

b) $\dfrac{1}{\tan x \csc x}$

To get rid of the fractions, we can do this
1) find common denominator for the ENTIRE expression

C Denominator: cosx sinx

2) multiply by CD "top" and "bottom"

c) $\dfrac{1-\cot x}{\tan x - 1} = \dfrac{\left(1 - \dfrac{\cos x}{\sin x}\right)\cos x \sin x}{\left(\dfrac{\sin x}{\cos x} - 1\right)\cos x \sin x}$

$\dfrac{1(\cos x \sin x) - \left(\dfrac{\cos x}{\cancel{\sin x}}\right)\dfrac{(\cos x \cancel{\sin x})}{1}}{\left(\dfrac{\sin x}{\cancel{\cos x}}\right)(\cancel{\cos x}\sin x) - 1(\cos x \sin x)} = \dfrac{\cos x \sin x - \cos^2 x}{\sin^2 x - \cos x \sin x}$

$= \dfrac{\cos x\,(\cancel{\sin x - \cos x})}{\sin x\,(\cancel{\sin x - \cos x})}$

$= \cot x$

d) $\dfrac{1+\cot^2 x}{\cot^2 x}$

e) $\sec x \cos x + \dfrac{\cos^2 x}{\sin^2 x}$

f) $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x}$

2. Verify the following identity algebraically, for $x = \frac{\pi}{4}$

$\dfrac{1-\sin x}{\cos x} = \dfrac{\cos x}{1+\sin x}$

3. Write each expression as a single trigonometric function.

a) $\cos 20^\circ \cos 5^\circ + \sin 20^\circ \sin 5^\circ$

b) $2\cos^2\left(\frac{\pi}{5}\right) - 1$

c) $2\sin 7x \cos 7x$

C_15 Ch 6 Hand-in 2022

Chapter 6 Hand-in Assignment – Trigonometric Identities

Name: Key

1. Simplify each expression.

a) $\dfrac{\tan x}{\sec x} = \dfrac{\dfrac{\sin x}{\cos x}}{\dfrac{1}{\cos x}} = \dfrac{\sin x}{\cancel{\cos x}} \cdot \dfrac{\cancel{\cos x}}{1} = \boxed{\sin x}$

b) $\dfrac{1}{\tan x \csc x} \quad \dfrac{1}{\dfrac{\cancel{\sin x}}{\cos x} \cdot \dfrac{1}{\cancel{\sin x}}} = \dfrac{1}{\dfrac{1}{\cos x}} = \dfrac{1}{1} \times \dfrac{\cos x}{1} = \boxed{\cos x}$

c) $\dfrac{1-\cot x}{\tan x - 1} \quad \dfrac{\left(1 - \dfrac{\cos x}{\sin x}\right)\sin x \cos x}{\left(\dfrac{\sin x}{\cos x} - 1\right)\sin x \cos x} = \dfrac{\sin x \cos x - \dfrac{\cos x}{\cancel{\sin x}} \cdot \cancel{\sin x} \cos x}{\dfrac{\sin x}{\cancel{\cos x}} \cdot \sin x \cancel{\cos x} - \sin x \cos x} = \dfrac{\sin x \cos x - \cos^2 x}{\sin^2 x - \sin x \cos x} = \dfrac{\cos x\,(\cancel{\sin x - \cos x})}{\sin x\,(\cancel{\sin x - \cos x})}$

$= \dfrac{\cos x}{\sin x}$

$= \boxed{\cot x}$

d) $\dfrac{1+\cot^2 x}{\cot^2 x} = \dfrac{\csc^2 x}{\dfrac{\cos^2 x}{\sin^2 x}} = \dfrac{\dfrac{1}{\sin^2 x}}{\dfrac{\cos^2 x}{\sin^2 x}} = \dfrac{1}{\cancel{\sin^2 x}} \cdot \dfrac{\cancel{\sin^2 x}}{\cos^2 x} = \dfrac{1}{\cos^2 x} = \boxed{\sec^2 x}$

e) $\sec x \cos x + \dfrac{\cos^2 x}{\sin^2 x} = \dfrac{1}{\cancel{\cos x}} \cdot \cancel{\cos x} + \cot^2 x$

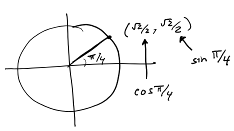
$= 1 + \cot^2 x$

$= \boxed{\csc^2 x}$

f) $\dfrac{1}{\sec^2 x} + \dfrac{1}{\csc^2 x} = \dfrac{1}{\dfrac{1}{\cos^2 x}} + \dfrac{1}{\dfrac{1}{\sin^2 x}} = 1 \times \dfrac{\cos^2 x}{1} + 1 \times \dfrac{\sin^2 x}{1}$

$= \cos^2 x + \sin^2 x$

$= \boxed{1}$



2. Verify the following identity algebraically, for $x = \frac{\pi}{4}$

$\dfrac{1-\sin x}{\cos x} = \dfrac{\cos x}{1+\sin x}$

$\dfrac{\left(1 - \frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} \stackrel{?}{=} \dfrac{\left(\frac{\sqrt{2}}{2}\right)}{\left(1 + \frac{\sqrt{2}}{2}\right)}$

0.414...	0.414...

Same value ⇒ verified

3. Write each expression as a single trigonometric function.

a) $\cos 20^\circ \cos 5^\circ + \sin 20^\circ \sin 5^\circ$

$\boxed{\text{Using } \cos A \cos B + \sin A \sin B = \cos(A-B)}$

$= \cos(20^\circ - 5^\circ)$

$= \cos 15^\circ$

b) $2\cos^2\left(\frac{\pi}{5}\right) - 1$

$\boxed{\text{Using } \cos 2\theta = 2\cos^2\theta - 1}$

$= \cos\left(2 \cdot \frac{\pi}{5}\right)$

$= \cos\left(\frac{2\pi}{5}\right)$

c) $2\sin 7x \cos 7x$

$\boxed{\text{Using } \sin 2\theta = 2\sin\theta\cos\theta}$

$= \sin(2 \cdot 7x)$

$= \sin(14x)$

4. Use identities and special angle values to determine the exact value of each trigonometric expression.

a) $\cos 195^\circ$

b) $\sin 255^\circ$

5. If $\angle A$ is in quadrant I, $\angle B$ is in quadrant III, and $\sin A = \frac{7}{25}$ and $\cos B = -\frac{8}{17}$, use identities to evaluate each of the following:

a) ~~[redacted]~~ $\sin(A-B)$

b) $\cos 2B$

c) $\sin 2A$

4. Use identities and special angle values to determine the exact value of each trigonometric expression.

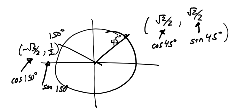
a) $\cos 195^\circ$ $\quad$ $\boxed{\text{Using } \cos(A+B) = \cos A\cos B - \sin A \sin B}$

$= \cos(150^\circ + 45^\circ)$

$= \cos 150^\circ \cos 45^\circ - \sin 150^\circ \sin 45^\circ$

$= \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \boxed{\frac{-\sqrt{6}-\sqrt{2}}{4}}$



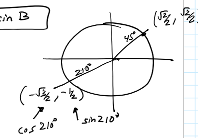
b) $\sin 255^\circ$ $\quad$ $\boxed{\text{Using } \sin(A+B) = \sin A\cos B + \cos A \sin B}$

$= \sin(210^\circ + 45^\circ)$

$= \sin 210^\circ \cos 45^\circ + \cos 210^\circ \sin 45^\circ$

$= \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \boxed{\frac{-\sqrt{2}-\sqrt{6}}{4}}$

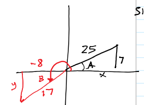


5. If $\angle A$ is in quadrant I, $\angle B$ is in quadrant III, and $\sin A = \frac{7}{25}$ and $\cos B = -\frac{8}{17}$, use identities to evaluate each of the following:

a) ~~[redacted]~~ $\sin(A-B) = \sin A\cos B - \cos A \sin B$

$= \left(\frac{7}{25}\right)\left(\frac{-8}{17}\right) - \left(\frac{24}{25}\right)\left(\frac{-15}{17}\right)$

$= \frac{-56}{425} - \left(\frac{-360}{425}\right) = \boxed{\frac{304}{425}}$



$\sin A = \frac{7}{25}$

$x^2 + 7^2 = 25^2$

$x^2 = 625 - 49$

$x^2 = 576$

$x = \pm 24$, use $x = 24$

$\cos A = \frac{24}{25}$

$\cos B = \frac{-8}{17}$

$(-8)^2 + y^2 = 17^2$

$64 + y^2 = 289$

$y^2 = 225$

$y = \pm 15$ use $y = -15$

$\sin B = \frac{-15}{17}$

b) $\cos 2B = \cos^2 B - \sin^2 B$

$= (\cos B)^2 - (\sin B)^2$

$= \left(\frac{-8}{17}\right)^2 - \left(\frac{-15}{17}\right)^2$

$= \frac{64}{289} - \frac{225}{289} = \boxed{\frac{-161}{289}}$

c) $\sin 2A = 2\sin A\cos A$

$= \left(\frac{2}{1}\right)\left(\frac{7}{25}\right)\left(\frac{24}{25}\right)$

$= \boxed{\frac{336}{625}}$

6. Prove the following identities:

a) $\frac{1+\cos 2x}{\sin 2x} = \cot x$

b) $\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = \frac{1}{1+\sec x}$

6. Prove the following identities:

a) $\frac{1+\cos 2x}{\sin 2x} = \cot x$

$\frac{1 + (2\cos^2 x - 1)}{2\sin x \cos x}$

$\frac{\cancel{1} + 2\cos^2 x - \cancel{1}}{2\sin x\cos x}$

$\frac{\cancel{2}\cos^{\cancel{2}} x}{\cancel{2}\sin x \cancel{\cos x}}$

$\frac{\cos x}{\sin x}$

$\cot x$

b) $\frac{1-\sin^2 x - 2\cos x}{\cos^2 x - \cos x - 2} = \frac{1}{1+\sec x}$

$\frac{\cos^2 x - 2\cos x}{(\cos x + 1)(\cos x - 2)}$ $\quad$ $\left(\frac{1}{1 + \frac{1}{\cos x}}\right) \frac{\cos x}{\cos x}$

$\frac{\cos x\,\cancel{(\cos x - 2)}}{(\cos x + 1)\cancel{(\cos x - 2)}}$ $\quad$ $\frac{\cos x}{\cos x + \frac{\cos x}{\cos x}}$

$\frac{\cos x}{\cos x + 1}$ $\quad$ $\frac{\cos x}{\cos x + 1}$

c) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

d) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

c) $\dfrac{\csc x + \cot x}{\tan x + \sin x} = \cot x \csc x$

$$\frac{\frac{1}{\sin x} + \frac{\cos x}{\sin x}}{\frac{\sin x}{\cos x} + \sin x} \cdot \frac{\sin x \cos x}{\sin x \cos x}$$

$$\frac{\frac{\cancel{\sin x}\cos x}{\cancel{\sin x}} + \frac{\cancel{\sin x}\cos^2 x}{\cancel{\sin x}}}{\frac{\sin^2 x\cancel{\cos x}}{\cancel{\cos x}} + \sin^2 x\cos x}$$

$$\frac{\cos x + \cos^2 x}{\sin^2 x + \sin^2 x \cos x}$$

$$\frac{\cos x(1 + \cancel{\cos x})}{\sin^2 x(1 + \cancel{\cos x})}$$

$$\frac{\cos x}{\sin^2 x}$$

$$\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x}$$

$$\frac{\cos x}{\sin^2 x}$$

d) $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$

$$\frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

$$\frac{1 + \sin x}{\cos x}$$

$$\left(\frac{\cos x}{1 - \sin x}\right)\frac{(1 + \sin x)}{(1 + \sin x)}$$

Multiply by Conjugate.

$$\frac{\cos x\,(1 + \sin x)}{1 + \cancel{\sin x} - \cancel{\sin x} - \sin^2 x}$$

$$\frac{\cos x\,(1 + \sin x)}{1 - \sin^2 x}$$

$$\frac{\cancel{\cos x}\,(1 + \sin x)}{\cos^{\cancel{2}} x}$$

$$\frac{1 + \sin x}{\cos x}$$

7. Solve each equation algebraically, over the given domain. You will need to use identities!

a) ~~$\cos x \tan x - \sin^2 x = 0$~~, for $0° \le x < 360°$.

$\sin 2x + \cos x = 0$

b) $\sin^2 x = \cos x - \cos 2x$, for $0 \le x < 2\pi$

7. Solve each equation algebraically, over the given domain. You will need to use identities!

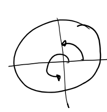
a) ~~$\cos x \tan x - \sin^2 x = 0$~~, for $0° \le x < 360°$. answers in degrees

$\sin 2x + \cos x = 0$

$2 \sin x \cos x + \cos x = 0$

$\cos x\,(2 \sin x + 1) = 0$

$\cos x = 0$

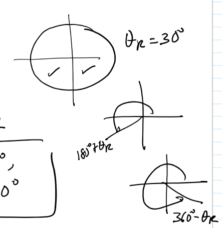


$\boxed{x = 90°,\ 270°}$

$2 \sin x + 1 = 0$

$2 \sin x = -1$

$\sin x = -\frac{1}{2}$



$\boxed{x = 210°,\ 330°}$

b) $\sin^2 x = \cos x - \cos 2x$, for $0 \le x < 2\pi$ use radians for answers.

$1 - \cos^2 x = \cos x - (2\cos^2 x - 1)$

$\cancel{1} - \cos^2 x = \cos x - 2\cos^2 x + \cancel{1}$

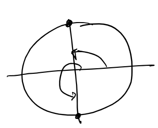
Use identities to get equation all in terms of $\cos(x)$.

$2\cos^2 x - \cos^2 x - \cos x = 0$

$\cos^2 x - \cos x = 0$

$\cos x\,(\cos x - 1) = 0$

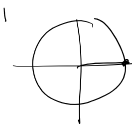
$\cos x = 0$



$\boxed{x = \pi/2,\ 3\pi/2}$

$\cos x - 1 = 0$

$\cos x = 1$



$\boxed{x = 0}$